SIMULATING SIN CITY

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ABSTRACT

We are interested in analyzing the probability that a casino in Las Vegas goes bankrupt. This work is concerned with utilizing a novel Monte Carlo based estimator of the ruin probability within the frame of a given risk model. This new estimator out-performs the classical Monte Carlo estimator in computing time and accuracy. Using the new estimator, sensitivity analysis is then conducted in order to understand the effect of changing the parameters of the risk model on the likelihood of bankruptcy.

INTO TO RUIN THEORY

Consider a Casino with:

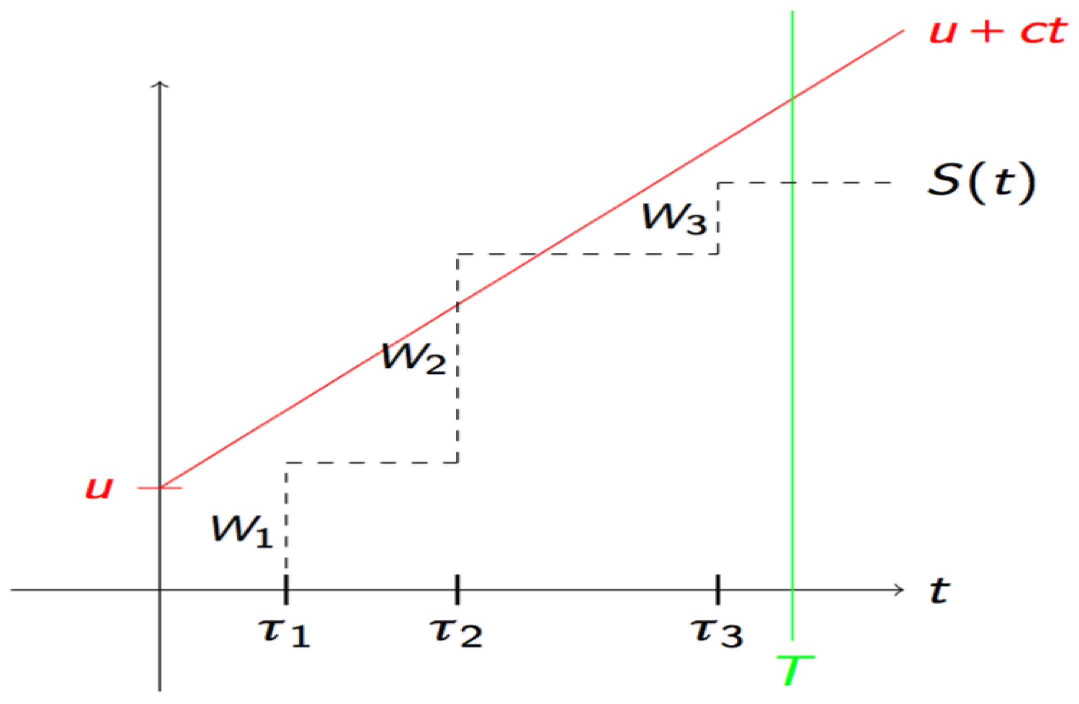
- Initial reserve u
- The number of slot machine winners up to time t is distributed as a Poisson process $\{N(t), t \geq 0\}$
- The jackpot sizes form a sequence of non-negative iid random variables $\{W_i, i \geq 1\}$
- Money collected linearly in time at rate c
  - $c = (1+\eta)E[N(1)]E[W_i]$
  - η : safety loading

The surplus at time t is given by:

$$R(t) = u + ct - \sum_{k=1}^{N(t)} W_i$$

RUIN VISUALIZATION

- Ruin time: $\tau_u = \min\{t \geq 0 \mid R(t) \leq 0\}$
- Non-Ruin probability: $\Phi(u, T) = \mathbb{P}(\tau_u > T)$



Risk Process Visualization

- In this example $\tau_u = \tau_2$, which is the first time $S(t)$ reaches the red line $(u + ct)$.

MONTE CARLO SIMULATION

We can evaluate the non-ruin probability using two numerical methods: Crude Monte Carlo (CMC) and Appell Polynomial Monte Carlo (APMC). The APMC method relies on the Appell Polynomial $A_n(1|U)$, which represents the probability that no ruin has occurred, given $N(t) = n$. U corresponds to the times at which the surplus reaches n.

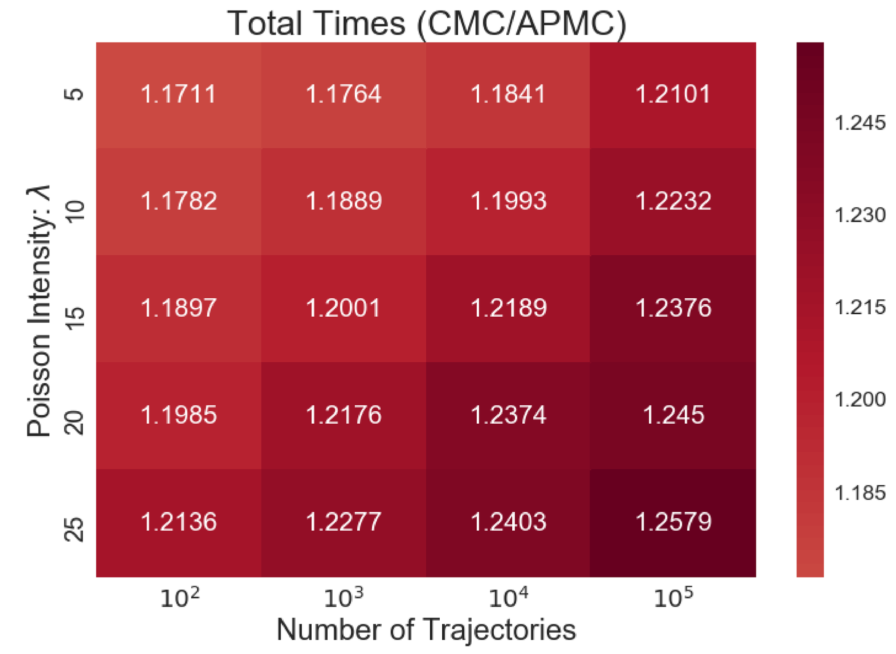
CMC

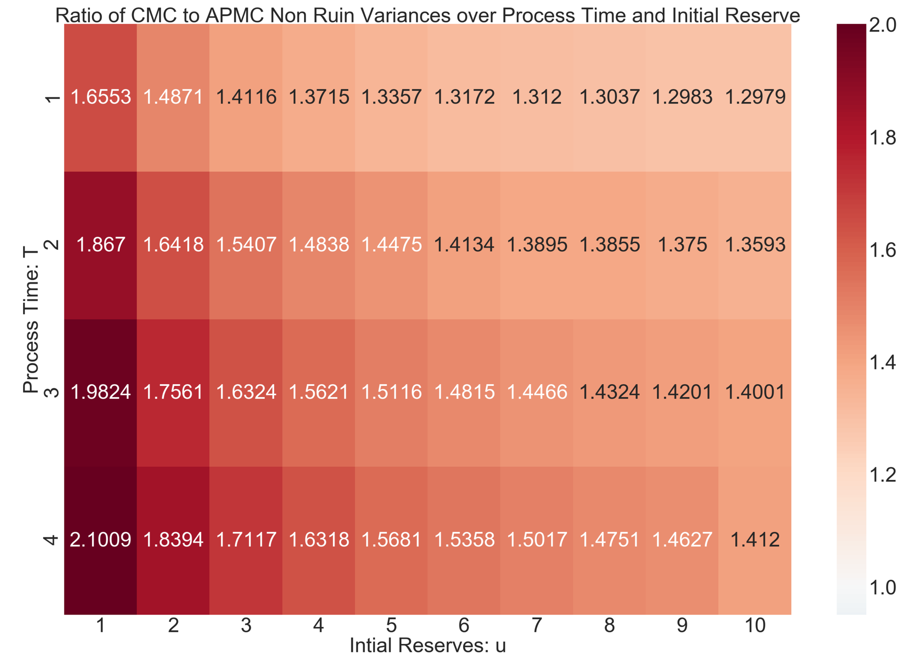
- Brute force method
- Creates process many times:
  - Records 1 if S(t) ever crosses u+ct
  - Records 0 if S(t) never crosses u+ct
- Non-ruin probability is average of our values

APMC

- Implements recursive analysis
- Creates process many times:
  - Analyzes Appell Polynomial $A_n(1|U)$
- Non-ruin probability is average of our Appell Polynomials

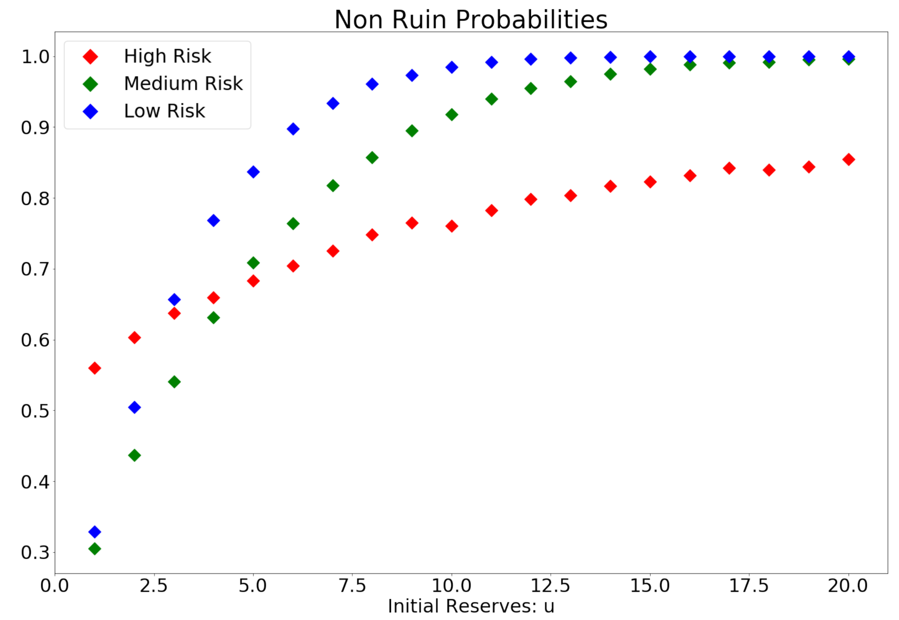
CMC V APMC

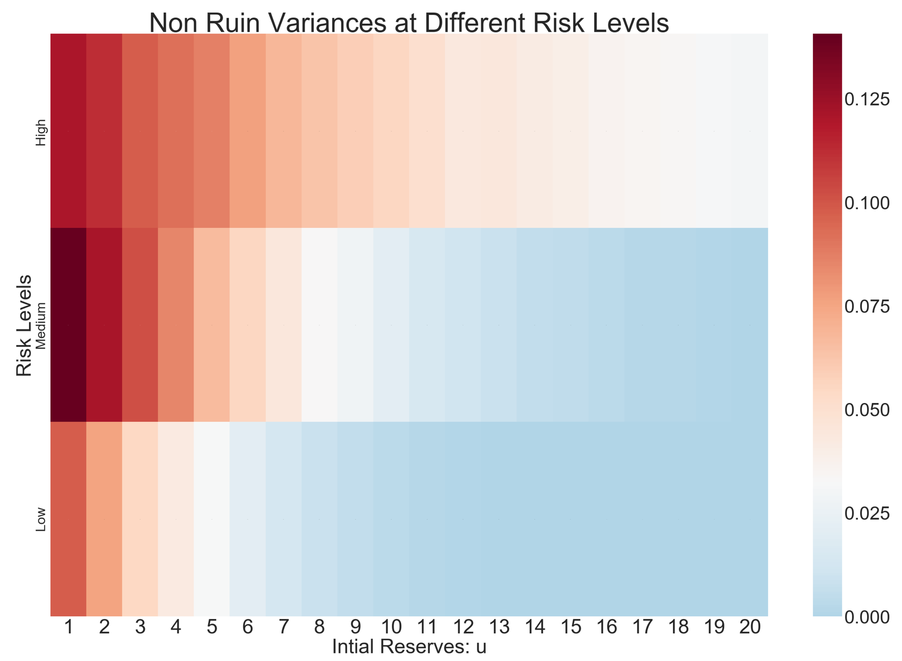




The first table depicts the CMC process time divided by APMC process time. All the values are greater than 1, indicating that the APMC is always faster than the CMC, with the difference growing as we increase in process intensity and number of trajectories. The second table portrays the CMC variance divided by the APMC variance. Again, all values are greater than 1, illustrating that the APMC is always more accurate than the CMC, with the difference expanding as we increase in time and decrease initial reserves. Thus, the APMC method is superior and we implement it to carry out further analysis.

SENSITIVITY ANALYSIS





We adjusted the winnings distributions of the slot machines by changing how often jackpots happened and how large each jackpot was. We categorized the distributions by risk: high, medium, and low. The slot machines with the greatest variance were least resistant to changes in initial reserves. This means that for a casino to be successful, they have to decrease the variance of their slot machine winnings.

CONCLUSION

The APMC method is superior to the CMC method, with respect to computation time and accuracy, for analyzing slot machine winnings for a casino. Furthermore, casinos can protect themselves by decreasing the variance of their slot machine winnings

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