1. The statistical consultant analyzed a time series of 50 observations and proposed a model of the form: \( X_t - \mu = Z_t - \theta Z_{t-1} \). The consultant believes that \( \theta > 0.9 \). Is the following statement compatible with the proposed model?

\[
\sum_{k=1}^{5} \hat{\rho}_W^2(k) = 0.005
\]

2. In modeling the weekly sales of a certain commodity over the past six months, the time series model \( X_t - \phi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1} \) was thought to be appropriate. Suppose the model was fitted and the autocorrelations of the residuals were:

\[
\begin{array}{cccccccc}
  k & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  \hat{\rho}_W(k) & .50 & -.04 & .03 & -.01 & .01 & .02 & .03 & -.01 \\
  \text{st. dev } \hat{\rho}_W(k) & .08 & .10 & .11 & .11 & .11 & .11 & .11 & .11 \\
\end{array}
\]

Is the assumed model really appropriate? If not, how would you modify the model? Explain.

3. You fit 100 observations, using ARMA(1,2) model. For the 100 residuals, you have determined the first seven autocorrelation coefficients:

\[
\begin{array}{cccccccc}
  k & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
  \hat{\rho}_W(k) & .20 & .15 & -.18 & .16 & .08 & .07 & .09 \\
\end{array}
\]

A \( \chi^2 \) test was performed to test adequacy of the model. Calculate the \( \chi^2 \) value and the number of degrees of freedom.

4. Two hundred observations of a time series, \( X_1, \ldots, X_{200} \), gave the following sample statistics:

sample mean: \( \bar{x}_{200} = 3.82 \)

sample variance: \( \hat{\gamma}(0) = 1.15 \)

sample ACF: \( \hat{\rho}(1) = .427, \hat{\rho}(2) = .475, \hat{\rho}(3) = .19 \)

(a) Based on these sample statistics, is it reasonable to suppose that \( \{X_t - \mu\} \) is white noise?

(b) Assuming that \( \{X_t - \mu\} \) can be modeled as the AR(2) process,

\[
X_t - \mu - \phi_1 (X_{t-1} - \mu) - \phi_2 (X_{t-2} - \mu) = Z_t,
\]

where \( Z_t \sim \text{IID}(0, \sigma_Z^2) \), find estimates of \( \mu, \phi_1, \phi_2, \) and \( \sigma_Z^2 \).

(c) Assuming that the data was generated from an AR(2) model, derive estimates of the PACF for all lags \( h \geq 1 \).