1. (a) Define a time series.
   (b) Give examples of time series likely to be stationary and examples of time series likely to be nonstationary.

2. Evaluate ACF function $\rho_X(k), k = 0, 1, \ldots$ for
   (a) $X_t = Z_t + \theta Z_{t-1}, Z \sim WN(0, \sigma_Z^2)$.
   (b) $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}, Z \sim WN(0, \sigma_Z^2)$.

3. Which of the following processes are stationary?
   (a) $X_t = 0.3 + 0.2Z_t + Z_{t-2}$
   (b) $X_t = 1 + 0.5Z_0$
   Here $\{Z_t\}$ is a Gaussian white noise, i.e. $\{Z_t\}$ is a sequence of i.i.d. normal r.v.s each with mean zero and variance $\sigma_Z^2$.

4. Given MA(2) $X_t = Z_t - 0.2Z_{t-1} - 0.7Z_{t-2}$, find $\rho(2)$.

5. Given the time series $X_t = Z_t - \theta Z_{t-1}$ and

   \[
   \frac{EX_tX_{t+1}}{EX_t^2} = -0.4
   \]

   determine $\theta$, so that the model is invertible.

Problems 6 and 7 are for students enrolled in PSTAT 274 ONLY

6. Let $\{X_t\}$ be MA(2) process given by $X_t = Z_t + \theta Z_{t-2}$ where $\{Z_t\} \sim WN(0, 1)$.
   (a) Find the autocovariance and autocorrelation function for this process when $\theta = 0.8$.
   (b) Compute the variance of the sample mean $(X_1 + X_2 + X_3 + X_4)/4$ when $\theta = 0.8$.
   (c) Repeat (b) when $\theta = -0.8$ and compare your answer with the result obtained in (b).

7. Let $\{Z_t\}$ be Gaussian white noise, i.e. $\{Z_t\}$ is a sequence of i.i.d. normal r.v.s each with mean zero and variance 1. Define

   \[
   X_t = \begin{cases} 
   Z_t, & \text{if } t \text{ is even;} \\
   (Z_{t-1}^2 - 1)/\sqrt{2}, & \text{if } t \text{ is odd}
   \end{cases}
   \]

   Show that $\{X_t\}$ is WN(0, 1) but that $X_t$ and $X_{t-1}$ are not i.i.d.