1. Prove that
\[ \int_0^t B_s^2 \, dB_s = \frac{1}{3} B_t^3 - \int_0^t B_s \, ds. \]

2. Let \( B \) be standard Brownian motion. Prove or disprove that \( U \) is a martingale, where
\[ U_t := B_t^2 - t. \]
Is the family \( \{ U_t, \ t \geq 0 \} \) uniformly integrable?

3. Let \( Y \) be a real valued RV on some probability space and assume that \( E|Y| < \infty \). Let \( \{ F_t \} \) be a filtration. Prove or disprove that \( V \) is a martingale (with respect to \( \{ F_t \} \)), where
\[ V_t := E(Y \mid F_t), \ t \geq 0. \]

4. A S.P. is called \textit{continuous in mean square} if
\[ \lim_{s \to t} E[(B_s - B_t)^2] = 0, \ \forall t \geq 0. \]
Is standard Brownian motion continuous in mean square? Why?

GOOD LUCK!