1. Using the notation as in class, let $S_n = \sum_1^n \xi_i$ denote the $n$-th step of the random walk with i.i.d. increments, where the increment $\xi_i$ satisfy $E|\xi_i| < \infty$ and $E\xi_i = 0$. Define

\[ M_n := S_n^2 - nE(\xi_1^2). \]

Prove that $M$ is a martingale with respect to the canonical filtration.

2. Let $X$ be a stochastic process on $\mathbb{R}^d$ with a.s. continuous paths, adapted to some filtration $\{\mathcal{F}_t\}$. Let $H_\Gamma$ be the hitting time of $\Gamma$, where $\Gamma$ is an open subset of $\mathbb{R}^d$. Prove that $H_\Gamma$ is an optional time for the same filtration.

3. Show that nonnegative constants (as random variables) are stopping times.

4. Show that every stopping time is also an optional time, and show that the converse is not true.

GOOD LUCK!