An Exploration of Systemic Risk in Random Banking Networks

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Abstract

We study the probability of a systemic event occurring within a banking network through different levels of connectivity between banks and the probability of multiple banks defaulting. We accomplish this using a mathematical model that connects the structure of banking networks to systemic risk. In particular, we are interested in the non-negligible tail of the loss distribution and how this tail is impacted by changes in the model parameters. We begin by interpreting a stochastic model that calculates the effects of a small change in the wealth of each bank and we advance the model to include an additional element of randomness to capture the variability of lending and borrowing in a financial system.

Main Objectives

1. Study the distribution of systemic events (via the number of defaults).
2. Investigate the relation between the model's parameters and the distribution of defaults.
3. Generalize Fouque & Sun's [1] model by incorporating the Erdos-Renyi random graph.

Methods

The model describes the effects of a small change in the wealth of a particular bank. We simulate the banks' wealth under the model dynamics by using Euler's Method [5] to discretize the stochastic differential equation (SDE). As model parameters change, we use the distribution of default to study systemic risk events.

The Model

We begin by recreating and analyzing a model put forth in a paper titled *Systemic Risk Illustrated* by Jean-Pierre Fouque & Li-Hsien Sun [1]; in which the wealth of each bank in given by:

$$dY_t^{(i)} = \frac{\alpha}{N}\sum_{j=1}^{N}(Y_t^{(j)} - Y_t^{(i)})dt + \sigma dW_t^{(i)}, i = 1, ..., N, \tag{1}$$

· α : level of connectivity

· N : number of banks

· $\sum_{j=1}^{N}(Y_t^{(j)} - Y_t^{(i)})dt$: drift term

· σ : diffusion coefficient

· $dW_t^{(i)}$: independent Brownian Motion

We then create a generalized model that incorporates a random link between banks with probability p:

$$dY_t^{(i)} = \frac{\alpha}{\sum_{j=1}^{N} a_{ij}}\sum_{j=1}^{N} a_{ij}(Y_t^{(j)} - Y_t^{(i)})dt + \sigma dW_t^{(i)}, i = 1, ..., N. \tag{2}$$

· a_{ij} : representation of a connection between bank i & j, where $a_{ij} \sim Bernoulli(p)$

· $\sum_{j=1}^{N} a_{ij}$: total number of connections for bank i

Results

Loss Distributions

We compare the default distributions of the random networks (2) to the default distributions of an unconnected network to visualize tail probabilities of systemic risk events. We perform this analysis because we wish to explore how different levels of p affect the tail probabilities. We use 10^4 Monte Carlo simulations to create loss distributions for the random systems and compare them to the independent networks using the Binomial distribution. We know from Fouque & Sun that losses in the independent case are Binomial(N, p), where $p \approx 0.5$.

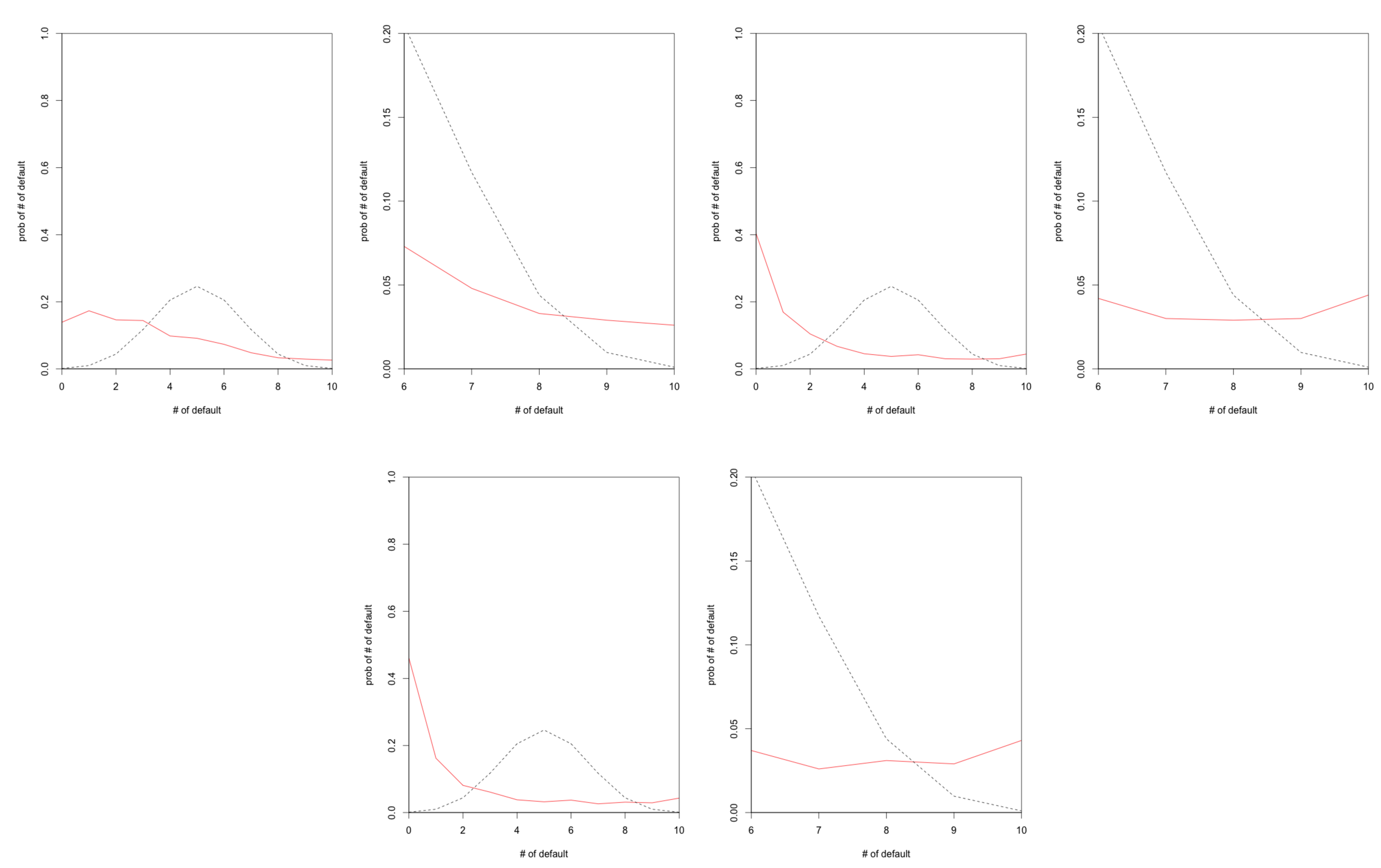

Figure 1: Plots of the loss distribution with $N = 10$, $\alpha = 10$, $\eta = -0.7$, and $p = 0.5$ (solid) and independent Brownian motions (dashed) to the left with corresponding tail probabilities to the right.

In these loss distributions we hold the levels of α, N and η to be constant at $\alpha = 10$, $N = 10$ and $\eta = -0.7$, and analyze how different levels of p manifest themselves in the default distributions of the coupled systems. At low levels of p we note that the default distributions of the coupled diffusions (solid lines) closely follow the default distributions of the uncoupled systems. However, as p increases there is a visible shift in the mass of defaults toward zero. This shows that increasing the probability of a connection between banks causes the system to mean revert because more banks are lending to and borrowing from each other.

Effect of N on Default Distributions

In this section, we hold the levels of p, α and η to be constant at $p = 1$, $\alpha = 10$ and $\eta = -0.7$, and vary number of banks N to analyze how systemic risk will be effected. We classify a systemic event to be the occurrence of at least half of the banks in the network defaulting. We analyze the 95% confidence intervals for this systemic event as N increases.

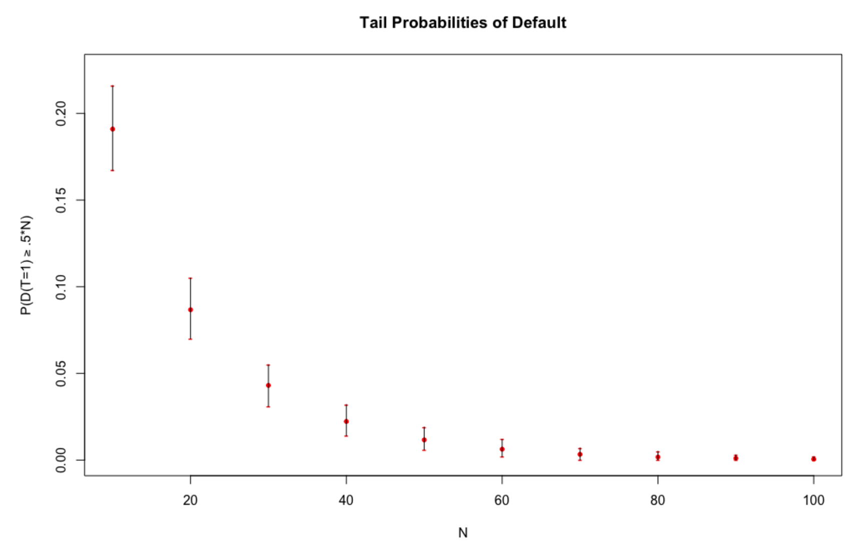

Figure 2: We show confidence intervals for the probabilities of at least 50% of banks defaulting for $N = 10, 20, ..., 100$.

As we increase the number of banks in the network, the probability of at least 50% of the banks defaulting decreases and levels off near zero. The widths of the confidence intervals decrease as we increase N, which highlights how the tail probabilities become less significant as the size of the network grows.

Effect of α on Default Distributions

In this section we hold the levels of p, N and η to be constant at $p = 1$, $N = 10$ and $\eta = -0.7$, and analyze how different levels of α impact the probability of a systemic event. Here, we determine the probability of every bank in the network defaulting in order to consider a systemic event.

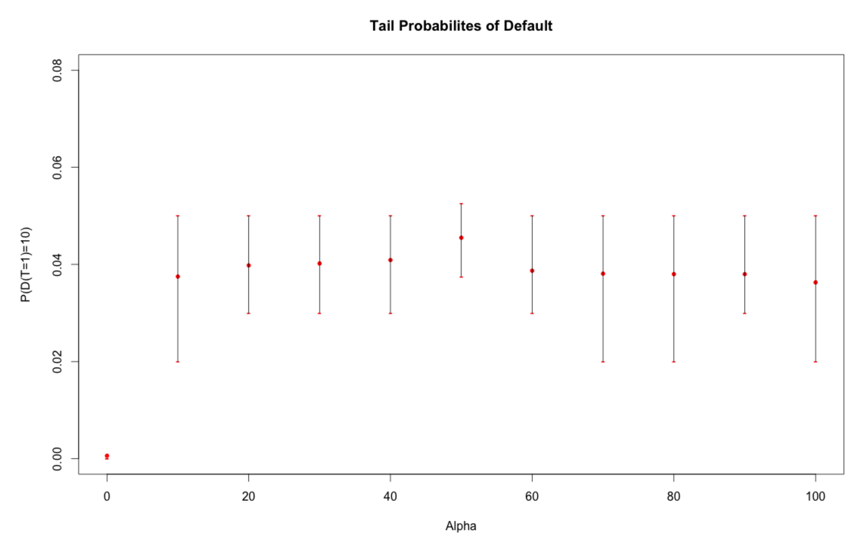

Figure 3: We show confidence intervals for the probabilities of 100% of banks defaulting for $\alpha = 10, 20, ..., 100$.

As we increase the level of connectivity between banks, the change in α fails to eliminate the systemic risk element. This result emphasizes that the non-negligible probability of systemic risk still occurs even as we increase the stability of the system.

Confidence Intervals for Average Wealth

We look at the impact of p to determine how far the sample paths deviate from the mean or average wealth. As we increase the probability of a connection between banks, we analyze three different confidence intervals: 50%, 90%, and 95%.

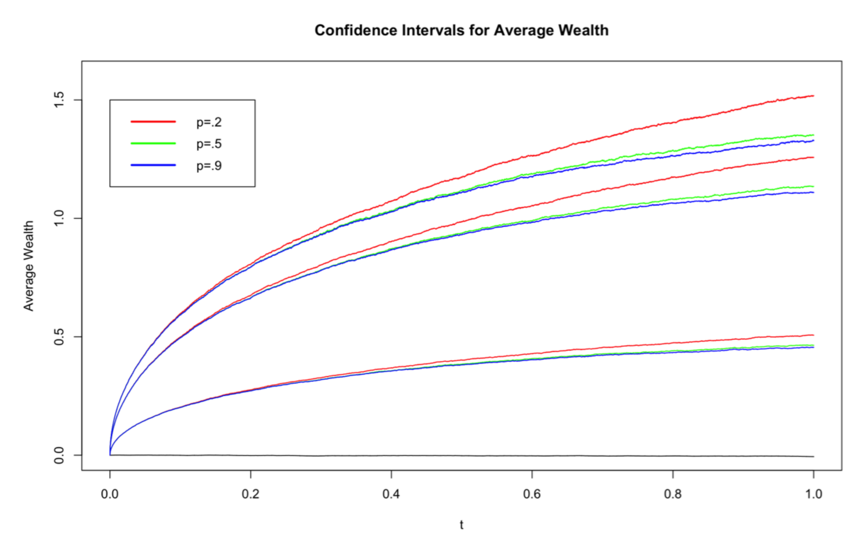

Figure 4: The top half of the 50%, 90%, and 95% confidence intervals (from bottom to top) for the $p = 0.2, 0.5,$ and 0.8.

When we focus on a particular confidence level, we notice that as p decreases the confidence interval width increases. This demonstrates the mean-reverting property of the model because as p approaches one more banks in the network are coupled.

Conclusion

- Systemic events in financial networks contribute to socio-economic dilemmas (i.e. social unrest, instability, unemployment).
- Probability of a systemic event remains non-negligible when stability increases.
- We created multiple graphs to visualize the behavior of systemic risk.
- Random graphs allow us to explore systems that more accurately reflect real-world network conditions.

References

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Acknowledgements

We would like to thank our advisor, Dr. Mackenzie Wildman for her assistance, guidance and enthusiasm. We would also like to thank the Center for Financial Mathematics and Actuarial Research for the opportunity to learn more about this fascinating topic.