Momentum Strategies using Reward-Risk Stock Selection Criteria

Svetlozar Rachev*

Chair-Professor, Chair of Econometrics, Statistics
and Mathematical Finance
School of Economics and Business Engineering
University of Karlsruhe,
Postfach 6980, 76128 Karlsruhe, Germany
and
Department of Statistics and Applied Probability
University of California, Santa Barbara
CA 93106-3110, USA
E-mail: rachev@statistik.uni-karlsruhe.de

Teo Jašic

Institute of Econometrics, Statistics
and Mathematical Finance
School of Economics and Business Engineering
University of Karlsruhe
Kollegium am Schloss, Bau II, 20.12, R210
Postfach 6980, D-76128, Karlsruhe, Germany
E-mail: teo.jasic@gmx.de

Stoyan Stoyanov

Chief Financial Researcher
FinAnalytica Inc.
130 S. Patterson
P.O. Box 6933
CA 93160-6933, USA
E-mail: stoyan.stoyanov@finanalytica.com

Frank J. Fabozzi

Frederick Frank Adjunct Professor of Finance,
Yale School of Management,
135 Prospect Street, Box 208200,
New Haven, Connecticut 06520-8200 U.S.A.
E-mail: fabozzi321@aol.com

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ABSTRACT

In this paper, we analyze momentum strategies that are based on reward-risk stock selection criteria in contrast to ordinary momentum strategies based on cumulative return criterion. Reward-risk stock selection criteria include standard Sharpe ratio with variance as a risk measure, and alternative reward-risk ratios with the expected shortfall as a risk measure. We investigate momentum strategies on 517 stocks in the S&P 500 universe in the period 1996 – 2003. Although the cumulative return criterion provides the highest average monthly momentum profits of 1.3% per month compared to the highest profit of 0.86% per month for the best alternative criterion, the alternative ratios provide better risk-adjusted returns measured on an independent risk-adjusted performance measure. We also provide evidence on unique distributional properties of extreme momentum portfolios analyzed within the framework of general non-normal stable distributions. Specifically, for every stock selection criterion, loser portfolios have the lowest tail index and tail index of winner portfolios is lower than that of middle deciles. The lower tail index is associated with lower mean strategy. The lowest tail index is obtained for cumulative return strategy. Given our data set, these findings indicate that cumulative return strategy obtains higher profits at the acceptance of the higher tail risk, while strategies based on reward-risk criteria obtain better risk-adjusted performance with the acceptance of the lower tail risk.

A number of studies document the profitability of momentum strategies across different markets and time periods (Jegadeesh and Titman, 1993, 2001, Rouwenhorst 1998, Griffin *et al.* 2003). The strategy of buying past winners and selling past losers over the time horizons between 6 and 12 months provides statistically significant and economically large payoffs with historically earned profits of about 1% per month. The empirical evidence on the momentum effect provides a serious challenge to asset pricing theory for several reasons. There is so far no consistent risk-based explanation and, contrary to other financial market anomalies such as the size and value effect that gradually disappear after discovery, momentum effect persists.

Stock selection criteria play a key role in momentum portfolio construction. While other studies apply simple cumulative return or total return criterion using monthly data, we apply reward-risk portfolio selection criteria to individual securities using daily data. A usual choice of reward-risk criterion is the ordinary Sharpe ratio corresponding to the static mean-variance framework. The mean-variance model is valid for investors if (1) the returns of individual assets are normally distributed, or (2) for a quadratic utility function, which indicates that investors always prefer portfolio with the minimum standard deviation for a given expected return. Either one of these assumptions are questionable. Regarding the first one, the empirical evidence invalidates the assumption of normally distributed asset returns since stock returns exhibit asymmetries. In addition, further distributional properties such as known kurtosis and skewness are lost in the one-period mean-variance approach.

Various measures of reward and risk can be used to compose alternative reward-risk ratios. We introduce alternative risk-adjusted criteria in the form of reward-risk ratios that use the expected shortfall as a measure of risk and expectation or expected shortfall as a measure of reward. The expected shortfall is an alternative to value-at-risk (VaR) measure that overcomes the limitations of VaR with regards to the properties of coherent risk measures (Arztner *et al.*, 1999). The motivation in using alternative risk-adjusted criteria is that they may provide strategies that obtain the same level of abnormal momentum returns but are less risky than those based on cumulative return criterion.

In previous and contemporary studies of momentum strategies, possible effects of non-normality of individual stock returns and their risk characteristics have not been explored in detail. Given abundant empirical evidence that stock returns exhibit non-normality, leptokurtic, and heteroscedastic properties, such effects are clearly important and may have a considerable impact on reward and riskiness of investment strategies. Extreme returns may occur with a much larger probability where the return distribution is heavy tailed than where it is normal. In addition, quantile-based measures of risk, such as value at risk (VaR), may also be significantly different if calculated for heavy-tailed distributions. Tokat *et al.* (2003) show that two distributional assumptions (normal and stable Paretian [Insert Footnote 1 here]) may result in considerably different asset allocations depending on the objective function and the risk-aversion level of the decision maker. By using the risk measures that pay more attention to the tail of the distribution, preserving the heavy tails with the use of a stable model makes an important difference to the investor who can earn up to a multiple of the return on the unit of risk he bears by applying the stable model. Thus, consideration of a non-normal return distribution plays an important role in the evaluation of the risk-return profile of individual stocks and portfolios of stocks.

We examine our strategies using various reward-risk criteria on the sample of 517 S&P 500 firms over the January 1996 – December 2003 period. The cumulative return criterion, Sharpe ratio and the best alternative ratio obtain monthly average returns of 1.3%, 0.68% and 0.86% respectively. We also evaluate the performance of different criteria using a risk-adjusted independent performance measure. On this measure, the best risk-adjusted performance is obtained using the best alternative ratio followed by the cumulative return criterion and the Sharpe ratio. Following our analysis, we argue that risk-adjusted momentum strategy using alternative ratios provides better risk-adjusted returns on risk-adjusted evaluation measure than cumulative return criterion although it provides lower momentum profits than those obtained using cumulative return criterion. Regarding the comparison of performance among various criteria, we find that the alternative criteria obtain better performance than the Sharpe ratio for our momentum strategy. A likely reason is that alternative ratios capture better the non-normality properties of stock returns and provide more precise estimation of the risk and reward than the traditional mean-variance measures such of the Sharpe ratio.

To analyze momentum effect in more detail, we perform the distributional analysis of momentum portfolio deciles. We examine whether there exists a systematic pattern of different parameters of the distribution across the momentum deciles formed on different criteria. Harvey and Siddique (2000) find a systematic skewness effect across momentum portfolio deciles in that the low expected return momentum portfolios have higher skewness than high expected return portfolios. Vorkink (2003) also provides evidence of a consistent pattern of skewness estimates across the momentum portfolios. Both of these studies use momentum strategy with the cumulative return criterion.

In our analysis, we extend the previous analyses in that we estimate the parameter of the stable distribution and examine the non-normal properties of the momentum deciles. Stable distributions are a class of probability laws that have interesting theoretical and practical properties. They generalize the normal (Gaussian) distribution and allow heavy tail and skewness, which are frequently seen in financial data. Our evidence shows that the loser portfolios have the lowest tail index for every criterion, that the tail index of the winner portfolio is higher than that of loser portfolio for every criterion. In addition, we also find a systematic skewness pattern across momentum portfolios for all criteria, with the sign and magnitude of the skewness differential between loser and winner portfolio dependent on the distribution parameter in reward-risk criteria. We interpret these findings as the evidence that extreme momentum portfolio returns have non-normal distribution and include additional risk component due to heavy tails. The part of abnormal momentum returns may be compensation for the acceptance of the positive tail risk and negative skewness differential between winner and loser portfolios.

The remainder of the paper is organized as follows. Section 1 provides a definition of risk-adjusted criteria and alternative reward-risk ratios. Section 2 describes our data and methodology. Section 3 conducts distributional analysis of the momentum portfolio daily returns obtained using different criteria and evaluates the performance of various momentum strategies using an independent risk-adjusted performance measure. Section 4 concludes the paper.

1. Risk-Adjusted Criteria for Stock Ranking

The usual approach to selecting winners and losers employed in previous and contemporary studies on momentum strategies has been to evaluate the individual stock's past monthly returns over the ranking period (e.g., six-month monthly return for the six-month ranking period). The realized cumulative return as a selection criterion is a simple measure, which does not include the risk component of the stock behavior in the ranking period [Insert Footnote 2 here]. To consider a risk component of an individual stock, we may apply different risk measures to capture the risk profile of stock returns. Variance or volatility of the stock returns captures the riskiness of the stock with regards to the traditional mean-variance framework. The disadvantage of variance is that the investor's perception of risk is symmetric around the mean. However, we consider other risk measures that overcome the deficiencies of the variance and satisfy the properties of coherent risk measures. In addition, empirical evidence shows that individual stock returns exhibit non-normality, so it would be more reliable to use a measure that could account for these properties.

Considering the non-normal properties of stock returns in the context of momentum trading, we aim to obtain risk-adjusted performance criterion that would be applicable to the most general case of a non-Gaussian stable distribution of asset returns [Insert Footnote 3 here]. It is a well established fact based on empirical evidence that asset returns are not normally distributed, yet the vast majority of the concepts and methods in theoretical and empirical finance assume that asset returns follow a normal distribution. Since the initial work of Mandelbrot (1963) and Fama and French (1963; 1965) who rejected the standard hypothesis of normally distributed returns in favor of more general stable Paretian distribution, the stable distribution has been applied to modeling both the unconditional and conditional return distributions, as well as theoretical framework of portfolio theory and market equilibrium models (see Rachev, 2003). While the stable distributions are stable under addition (i.e., a sum of stable independent and identically distributed (i.i.d.) random variables is a stable random variable), they are fat-tailed to that extent that their variance and all higher moments are infinite.

We introduce next the expected shortfall as a measure of risk that will be used as the risk component of our alternative reward-risk criteria.

Expected Shortfall

Usual measures of risk are standard deviation and value at risk (VaR). The VaR at level $(1-a)100\%$, $a\ ?\ [0,1]$, denoted $VaR_{(1-a)100\%}(r)$ for an investment with random return r, is defined by $Pr(l > VaR_{(1-a)100\%}(r)) = a$, where $l = -r$ is the random loss, that can occur over the investment time horizon. In practice, values of a close to zero are of interest, with typical values of 0.05 and 0.01. VaR is not "sensitive" to diversification and, even for sums of independent risky positions, its behavior is not as we would expect (Fritelli and Gianin, 2002). The deficiencies of the VaR measure prompted Arztner *et al.* (1999) to propose a set of properties any reasonable risk measure should satisfy. They introduce the idea of coherent risk measures, with the properties of monotonicity, sub-additivity, translation invariance, and positive homogeneity.

Standard deviation and VaR are not coherent measures of risk. In general, VaR is not subadditive and is law invariant in a very strong sense. On the other hand, the *expected shortfall*, also called *conditional VaR*, or *expected tail loss* (ETL) is a coherent risk measure (Arztner *et al.*, 1999; Rockafellar and Uryasev, 2002, Bradley and Taqqu, 2003). ETL is a more conservative measure than VaR and looks at how severe the average (catastrophic) loss is if VaR is exceeded [Insert Footnote 4 here]. Formally, the ETL is defined by

$$ETL_{a100\%}(r) = E(l|l > VaR_{(1-a)100\%}(r)), \qquad (1)$$

where r is the return over the given time horizon, and $l = -r$ is the loss. $ETL_{a100\%}(r)$ is also known as *Conditional VaR* [Insert Footnote 5 here], denoted by $CVaR_{(1-a)100\%}(r) = ETL_{a100\%}(r)$ (see Martin *et al*. 2003).

The ETL is a subadditive, coherent risk measure and portfolio selection with the expected shortfall can be reduced to a linear optimization problem (see Martin *et al*. 2003, and the references therein).

Alternative Reward-Risk Ratios

Various reward-risk performance measures and ratios have been studied in the literature. Recently, Biglova *et al.* (2004) provide an overview of such risk-reward performance measures and compare them based on the criterion of maximizing the final wealth over a certain time period. The results of their study support the hypothesis that alternative risk-return ratios based on the expected shortfall capture the distributional behavior of the data better than the traditional Sharpe ratio. In order to include the risk profile assessment and account for non-normality of asset returns, we apply the alternative Stable-Tail Adjusted Return ratio (STARR) and the Rachev (R) ratio [Insert Footnote 6 here] as the criteria in forming momentum portfolios. We analyze and compare the traditional Sharpe ratio with alternative STARR and R-ratios for various parameter values that define different level of coverage of the tail of the distribution. A summary of the three reward-risk ratios is provided below:

1. *Sharpe Ratio.* The Sharpe ratio (see Sharpe, 1994) is the ratio between the expected excess return and its standard deviation:

$$r(r) = \frac{E(r - r_f)}{s_{(r-r_f)}} \qquad (2)$$

where r_f is the risk-free asset and s_r is the standard deviation of *r*. For this ratio it is assumed that the second moment of the excess return exists.

2. *$CVaR_{(1\text{-}a)100\%}$ Ratio* (*$STARR_{(1\text{-}a)100\%}$ Ratio*). The $CVaR_{(1\text{-}a)100\%}$ ratio (also known as *$STARR_{(1\text{-}a)100\%}$ Ratio*, see Martin *et al.*, 2003) is the ratio between the expected excess return and its conditional value at risk:

$$r(r) = \frac{E(r - r_f)}{CVaR_{(1-a)100\%}(r - r_f)} =: \text{STARR}_{(1\text{-}a)100\%} \qquad (3)$$

where $CVaR_{(1\text{-}a)100\%}(r) = ETL_{a100\%}(r)$, see (1).

3. *Rachev Ratio* (*R-ratio*). The R-ratio with parameters *a* and *ß* is defined as:

$$r(r) = \frac{ETL_{a\,100\%}(r_f - r)}{ETL_{b\,100\%}(r - r_f)} = \text{:R-ratio}(a,\ ß) \qquad (4)$$

where a and $ß$ are in [0,1]. Here, if r is a return on a portfolio or asset, and $ETL_a(r)$ is given by (1). We analyze the R-ratio for different parameters a and $ß$. For example, R-ratio ($a = 0.01$, $ß = 0.01$), R-Ratio ($a = 0.05$, $ß = 0.05$), and R-ratio ($a = 0.09$, $ß = 0.9$). These parameters cover different part of the distribution opposite of the excess returns and excess returns.

The R-ratio simultaneously maximizes the level of return and gets insurance for the maximum loss. The R-ratio as given by (4) can be interpreted as the ratio of the expected tail return above the level, divided by the expected tail loss. In other words, this is a ratio that awards extreme returns adjusted for extreme losses. Note that the STARR ratio is the special case of the R-ratio since $CVaR_{(1-a)100\%} = ETL_{a100\%}$. For example, STARR(95%) = R-ratio(1, 0.05). When the parameters in the R-ratio are used in percentage form, they correspond the to $(1 - a)$ notation to ease the comparison with the STARR ratios. We analyze the R-ratio for different parameters a and $ß$, which we may optimally calibrate in the back-testing analysis.

After forming the portfolio of winners and losers based on ranking calculated values of specific reward-risk ratio criteria for considered stocks in the ranking period, we evaluate the performance of momentum strategy in the holding period. Specifically, we analyze average momentum (winner–loser) spread returns, final wealth value of the momentum portfolio and their risk-adjusted performance. Due to the definition of the reward-risk ratios and computational requirements, we use daily data for their calculation. Following the analysis of momentum profits and risk-adjusted performance, we identify the best performing ratios, which allow investors to pursue a profitable and risk-balanced momentum strategy.

2. Formation of Momentum Portfolios using Reward-Risk Stock Selection Criteria

2.1 Data and Methodology

Our sample consists of a total of 517 stocks included in the S&P 500 index in the period January 1, 1996 to December 31, 2003. We analyze the daily returns of stocks in the observed 8-year time period. Daily stock returns were calculated from the time series as

$$r_t = \ln \frac{P_t}{P_{t-1}}$$

in the observed period, where P_t is the (dividend adjusted) stock price at t.

The momentum strategy is implemented by simultaneously selling losers and buying winners at the end of the formation or ranking period, and holding the portfolio over the investment or holding period. The stock selection criterion determines winners and losers in the ranking period based on prior returns for cumulative return criterion or risk-return profile for risk-return (i.e., reward-risk) ratio criterion. The zero-investment, self-financing strategy generates momentum profits in the holding period. Such zero-investment strategy is applicable in international equity investment management practice given the regulations on proceeds from short-sales for investors (Bris *et al.*, 2004).

We consider momentum strategies based on the ranking and holding periods of 6 months (i.e., 6-month/6-month strategy or shortly 6/6 strategy). We rank the stocks by applying the risk-adjusted criteria to their daily returns in the period of 6 months. Therefore, for each month t, the portfolio held during the investment period, months t to t+5, is determined by performance over the ranking period, months t-6 to t-1. Following the usual convention, we rank the stocks in ascending order and assign them to one of the ten sub-portfolios (deciles). "Winners" are those stocks with the top 10% ranking period returns and "losers" are those stocks with the lowest 10% ranking period returns (with returns of at least 12 months before applying the risk-return ratio criterion to the prior 6-month daily returns.). Winner and loser portfolios are equally weighted at formation and held for 6 subsequent months of the holding period. We apply our strategy to non-overlapping 6-month investment horizons. This means that positions are held for 6-months, after which the portfolio is re-constructed (rebalanced).

3. Results

3.1 Summary Statistics of Momentum Portfolio Returns

For each stock-selection criterion, we report the average monthly returns aggregated from daily data [Insert Footnote 7 here]. Unlike other studies that report *t*-statistics for average monthly returns, we calculate Kolmogorov-Smirnov (K-S) statistics [Insert Footnote 8 here] for normal and stable model fit to account for non-normality in daily returns. We also report the results of the final wealth value of winner and loser portfolios and their difference. Table 1 shows the average monthly returns of winner and loser portfolios as well as of the zero-cost, winner-loser spread portfolios for our 6-month/6-month strategy for all considered reward-risk ratios and cumulative return criterion. The highest average winner-loser spread of 1.28% per month (corresponding to 15.36% annualized return) for the 6-month/6-month strategy arises for cumulative return, followed by 0.86% per month (10.32% annualized return) obtained by the R-ratio(91%,91%). Thus, the annualized differential return between cumulative return and the best alternative ratio amounts to 5.04%. The Sharpe ratio achieves a momentum profit of 0.71% per month (8.52% annualized return). The lowest monthly momentum return of 0.21% is obtained for the R-ratio(50%,95%) ratio. For the STARR ratios, the returns of winner minus loser portfolios fall within the range between 0.61% per month and 0.79% per month.

[Insert Table 1 here]

The R-ratio(91%,91%) or R-ratio(0.09, 0.09) obtains the best performance with regards to realized average momentum profits when compared to other reward-risk ratio criteria. As explained in the previous section, the R-ratio is the ratio between the expected tail loss of the opposite of excess return at a given confidence level and the ETL of the excess return at another confidence level [Insert Footnote 9 here]. The R-ratio(0.09, 0.09) measures the reward at the 91 percentile threshold level and the expected value of portfolio returns so that VaR(91%) has been exceeded and most of the tail risk captured. The R-ratio with these parameters seems to capture well the distributional behavior of the data which is usually a component of risk due to heavy tails. The R-ratio(0.01, 0.01) that captures the risk of the extreme tail (i.e., covered by the measures of $ETL_{1\%}$) provides the second best performance among the R-ratios.

The STARR(95%) ratio that captures tail risk at the 95 percentile threshold level provides the best result on realized momentum profits among the STARR ratios (0.79% per month). Medium tail risk is measured by the STARR(75%) criterion which obtains the second best momentum profit among the STARR ratios. The STARR(50%) ratio covers the entire downside risk and obtains the average return that is slightly lower than that of the STARR(75%) ratio. The difference of realized momentum spreads between the largest and the lowest realized spread obtained by the STARR ratios is 0.19% per month, which translates to yearly return differential of 2.27%.

The Sharpe ratio obtains approximately the half of the momentum profit of the cumulative return criterion. Compared to other reward-risk selection criteria except the R-ratio(50%,99%) and R-ratio(50%,95%), the momentum strategy using the Sharpe ratio obtains the lower momentum profits.

Additional descriptive statistics including measures of volatility, skewness and kurtosis for the momentum-sorted portfolio deciles are reported in Table 1. The measure of skewness, $\hat{S}$, is calculated as

$$\hat{S} = \frac{1}{T}\sum_{t=1}^{T}\frac{(r_t - \bar{r})^3}{\hat{s}^3} \tag{5}$$

and the measure of kurtosis, $\hat{K}$, as

$$\hat{K} = \frac{1}{T}\sum_{t=1}^{T}\frac{(r_t - \bar{r})^4}{3\hat{s}^4} - 1 \tag{6}$$

where T is the number of observations and $\hat{s}$ is the estimator of the second moment. Under the assumptions of normality, $\hat{S}$ and $\hat{K}$ would have mean zero asymptotic normal distribution with variances $6/T$ and $8/3T$, respectively.

For cumulative return, Sharpe ratio and STARR ratios, the volatility measure obtains the U-shape across the momentum deciles, consistent with finding in Fama and

French (1996). The volatility of winner and loser portfolios for these criteria is about the same with the slightly greater value for the loser portfolio for every criterion. For the R-ratios there is no clear pattern of volatility measure across momentum deciles. The highest volatility estimates for winner and loser portfolios are obtained for cumulative return criterion with values of 0.01871 and 0.0171 respectively. The lowest estimates are obtained for the R-ratios with values of 0.013 and 0.012 for winner and loser portfolio respectively.

The univariate statistics for the momentum deciles show a clear systematic pattern for measure of skewness across the momentum portfolio deciles. $\hat{S}$ is large and positive for the decile 1 portfolio (loser) and decrease to the small positive or negative value $\hat{S}$ of the decile 10 portfolio (winner). This decrease in values is not strictly monotonic and the nature of decrease of these values depends on the ratio criteria. The $\hat{K}$ measures are largest for the loser portfolio, and are higher than those for winner portfolio for every stock selection criteria.

Our results on skewness differential between loser and winner portfolios are consistent with findings of Harvey and Siddique (2000), who use a cumulative return strategy on NYSE/AMEX and Nasdaq stocks with five different ranking (i.e., 35 months, 23 months, 11 months, 5 months and 2 months) and six holding periods (i.e., 1 month, 3 months , 6 months, 12 months, 24 months and 36 months) over the period 1927- 1997. To illustrate the skewness effect, in Figure 1 we plot the skewness measure across momentum portfolios for cumulative return, Sharpe ratio and STARR(95%) criterion.

[Insert Figure 1 here]

Harvey and Siddique (2000) show that for every of their momentum strategy definitions, the skewness of the loser portfolio is higher than that of the winner portfolio. They conclude that the higher mean strategy is associated with lower skewness. Although we consider much shorter data set than that of Harvey and Siddique, our conclusions are similar for extended set of reward-risk stock selection criteria. For every stock selection criteria that we use on our data set, the skewness of the loser portfolio is higher than that of the winner portfolio. Across different criteria, the average skewness for loser portfolio is –0.0160, whereas the average skewness for winner portfolio is 0.3108.

In summary, the cumulative return criterion obtains the largest average monthly momentum profit among all criteria, but this strategy is inherently riskier than other strategies measured on volatility of extreme winner and loser portfolios. The volatility of winner and loser portfolios is of similar magnitude with the highest volatility obtained for cumulative return criterion. When measured on volatility, the strategies based on the Sharpe ratio and the STARR ratios are less risky that the strategy on cumulative return criterion. The R-ratios obtain the lowest volatility of winner and loser portfolios among all the criteria. We also provide evidence of a systematic skewness pattern across momentum portfolio deciles with positive skewness differential between loser and winner portfolio. The higher mean strategy is associated with lower skewness. The skewness differential between winner and loser portfolios obtained by the R-ratios is lower than that obtained by other ratios. Given our data set. the results imply that strategy criterion that obtains the higher average realized profits needs to accept higher volatility risk of extreme portfolios and negative skewness differential between winner and loser portfolios.

3.2 Final Wealth of Momentum Portfolios

For estimation of the final wealth of the momentum portfolio, we assume that the initial value of the winner and loser portfolio is equal to 1 and that the initial cumulative return, CR_0, is equal 0 at the beginning of the first holding period. We then obtain the total return of the winner and loser portfolio and their difference is the final wealth of the portfolio. Given continuously compounded returns, the cumulative return CR_n in each holding period, is given by

$$CR_k = CR_{k-1} + x'_{M(k)} r_k \tag{7}$$

where $x'_{M(k)}$ is the momentum portfolio and r_k is the vector of continuously compounded returns in the holding period k, $k = 1$. The total cumulative return at the end of the whole observed period is the sum of the cumulative returns of every holding period.

Table 2 reports the result on the final wealth of the momentum portfolios at the end of the observation period (end of the last holding period) for every stock selection criterion. In general, the relative rankings of the final wealth values of different momentum strategies reflect those of average monthly profits from Table 1.

[Insert Table 2 here]

Final wealth for our 6-month/6-month strategy is positive for all reward-risk ratio criteria and cumulative return criterion. The highest value of the final wealth for winner-loser spread is obtained for cumulative return. Among the ratio criteria, the highest value of 3.6614 is obtained for STARR(95%), followed by the R-ratio(91%,91%) with value 3.3876. The lowest values of the final wealth of momentum portfolio, are obtained for the the R-ratio(50%,99%) and R-ratio(50%,95%) with the values of 0.9404 and 0.5864 respectively.

Figure 2 plots the cumulative realized profits (accumulated difference between winner and loser portfolio return over the whole period) to the 6-month/6-month strategy for the cumulative return criterion, Sharpe ratio, and the R-ratio(91%,91%). The graph of the cumulative realized profits for the R-ratio(91%,91%) shows better performance than the graph for the Sharpe ratio given the value of the total realized return of the portfolio at the end of the observed period. The total realized return of the winner-loser portfolio for cumulative return is higher than the total realized return for two observed ratios.

[Insert Figure 2 here]

3.3 Distributional Analysis of Momentum Portfolios using Estimates of Stable Distribution

It is a well known fact supported by empirical evidence that financial asset returns do not follow normal distributions. Mandelbrot (1963) and Fama and French (1963; 1965) were the first to formally acknowledge this fact and build the framework of using stable distributions to model financial data. The excessively peaked, heavy-tailed and asymmetric nature of the return distribution made the authors reject the Gaussian hypothesis in favor of more general stable distributions, which can incorporate excess kurtosis, fat tails and skewness. Since this initial work, the stable distribution has been applied to modeling both the unconditional and conditional return distributions, as well as theoretical framework of portfolio the ory and market equilibrium models (Rachev and Mittnik, 2000).

The stable distribution is defined as the limiting distribution of the sum of (i.i.d.) variables. The class of all stable distributions can be described by four parameters: (a, b, m, s). The parameter a is the index of stability and must satisfy $0 < a \leq 2$. When $a = 2$ we have the Gaussian distribution. The parameter ß determines skewness of the distribution and is within the range [-1,1]. If ß = 0, the distribution is symmetric. If ß > 0, the distribution is skewed to the right and to the left if ß < 0. The location is described by μ and s is the scale parameter, which measures the dispersion of the distribution corresponding to the standard deviation in Gaussian distributions. Stable distributions allow for skewed distributions when $b \neq 0$ and fat tails relative to the Gaussian distribution when $a < 2$. Stable Paretian laws have fat tails meaning that extreme events have high probability relative to the normal distribution when $a < 2$. The most important parameters are a and ß since they identify two fundamental properties that are untypical of the normal distribution – heavy tails and asymmetry.

The a-stable, or, in short, S_a distribution, has in general, no closed-form expression for its probability density function, but can, instead, be expressed by its characteristic function. The definition of a stable random variable X states that for $a \in (0,2]$, $b \in [-1,1]$, and $m \in \Re$, X has the characteristic function of the following form

$$f_X(t) = \begin{cases} \exp\left\{-s^a \, |t|^a \left(1 - ib\,sign(t)\tan\frac{pa}{2} + imt\right)\right\} & if\; a \neq 1 \\ \exp\left\{-s \, |t| \left(1 + ib\,sign(t)\ln|t| + imt\right)\right\} & if\; a = 1 \end{cases} \qquad (8)$$

Then the stable random variable X is denoted by $X \sim S_a(s, b, m)$. In particular, when both the skewness and location parameters ß and μ are zero, X is said to be symmetric a-stable and denoted by $X \sim S\alpha S$. A stable distribution is therefore determined by the four key parameters: α (index of stability or tail weight) determines density's kurtosis with $0 < \alpha \leq 2$, β determines density's skewness with $-1 \leq \beta \leq 1$, σ is a scale parameter (in the Gaussian case, α =2 and $2\sigma^2$ is the variance) and μ is a location parameter. If 1 < $\alpha \leq 2$, then μ is the usual mean estimator.

We estimate the parameters of a stable distribution and approximate the stable density functions by applying a maximum likelihood estimation using Fast Fourier Transform (FFT). For details of the estimation procedure, see Rachev and Mittnik (2000). The application of computationally demanding numerical approximation method in estimation of stable distribution is necessary, while closed-form expressions for its probability density function in general do not exist.

We examine the momentum portfolio deciles obtained on different stock selection criteria to understand how the abnormal returns of deciles relate to various parameters of the stable distribution. The estimation results on the used sets of momentum portfolios are reported in Table 3. The index of stability has the lowest value for the loser portfolio for every criterion. The α value for winner portfolio is higher than that of loser portfolio for every criterion and has the second lowest value for some criteria. The values of the tail index for the loser decile are in the range [1.567, 1.568] and for the winner decile in the range [1.767, 1.768]. The lowest tail index of 1.5670 for the loser portfolio is obtained for cumulative return criterion and the largest for the R-ratio(99%,99%). The tail index of the loser portfolio for the Sharpe ratio and the STARR criteria is slightly larger than that of cumulative return. The highest tail index of extreme portfolios is obtained for the R-ratios. It seems that the higher mean strategy is associated with higher tail index. The acceptance of the positive tail risk leads to higher momentum returns. The smaller differentials in α between winner and loser portfolios for the Sharpe ratio and alternative ratio criteria leads to smaller momentum returns.

The estimates of the scale parameter σ are highest for the winner and loser deciles, with the value for the winner decile higher than that for the loser decile, for every criterion. This is similar to results on volatility measure across momentum portfolios. In addition, for every criteria, the estimate of skewness β of the loser portfolio is higher than that of the winner portfolio, consistent with the results of distributional analysis under the normal distribution assumption presented in the previous section. Thus, the higher mean strategy is associated with the higher tail index and lower skewness for every criterion. The results suggest that in momentum-based trading strategies using cumulative return and reward-risk ratio criteria, constructing zero-investment portfolio requires acceptance of negative skewness and positive tail index. For cumulative return criterion, the differential of skewness and the tail index between winner and loser portfolios is the largest, implying that compensation for skewness and tail risk is reflected in higher momentum profits. Use of reward-risk criteria reduces the level of skewness and tail risk and the realized momentum returns are lower than those obtained with cumulative return criterion.

[Insert Table 3 here]

The figure 3 shows the estimate of the tail index across the momentum deciles for cumulative return, Sharpe ratio and the R-ratio(91%,91%) criterion. It is evident that the extreme momentum deciles (winner and loser) have lower estimates than the middle deciles for every criterion.

[Insert Figure 3 here]

We find that for the return distribution of every momentum decile, the estimated values of the index of stability a are below 2 and that there is obvious asymmetry ($b \neq 0$). These two facts strongly suggest that the Gaussian assumption is not a proper theoretical distribution model for describing the momentum portfolio return distribution. In addition, the extreme momentum portfolios (winner and loser) show unique characteristics with regards to the tail weight and skewness estimate. The lower tail estimate of the extreme decile can be the result of the lower tail estimate of the individual stocks that constitute the decile or reflect the strong dependency between them.

To further investigate the non-normality of the momentum decile returns, we assess whether the Gaussian distribution hypothesis holds for momentum decile returns and compare it to the stable Paretian hypothesis. We assume that daily momentum decile observations are (i.i.d.). This test concerns the unconditional homoscedastic distribution model. For the normality tests, we employ two goodness-of-fit tests which are based on the whole empirical distribution, the Kolmogorov distance (KD) and the Anderson-Darling (AD) statistic. We compare the goodness-of-fit in the case of the Gaussian hypothesis and the more general stable Paretian hypothesis.

The Kolmogorov distance is computed according to

$$KD = \sup_{x \in R} |\hat{F}(x) - F(x)| \tag{9}$$

where $\hat{F}(x)$ is the empirical sample distribution and $F(x)$ is the cumulative density function (cdf) of the postulated simulated distribution and emphasizes the deviations around the median of the distribution. It is a robust measure in the sense that it focuses only on the maximum deviation between the sample and fitted distributions.

The *AD*-statistic is computed as follows

$$AD = \sup_{x \in R} \frac{|\hat{F}(x) - F(x)|}{\sqrt{F(x)(1 - F(x))}} \tag{10}$$

where $\hat{F}(x)$ is the empirical cumulative distribution and $F(x)$ the chosen cumulative model distribution. The AD statistic weighs discrepancies appropriately across the whole support of the distribution and is particularly important if one is interested in determining tails of return distributions. Thus, we compare the empirical cumulative distribution $\hat{F}$ of momentum decile residuals with either a simulated Gaussian or a non-Gaussian distribution *F*.

Table 4 presents the goodness-of-fit statistics for the fitted unconditional distributions of the momentum deciles. The better fit of the stable Paretian assumption over the Gaussian assumption for momentum deciles is confirmed by analyzing the computed values of KD and AD statistics. For every momentum decile and every criteria, the KD statistic in the stable Paretian case is below the KD statistic in the Gaussian case. The same conclusion applies to the AD statistic. The large difference between the AD statistic computed for the stable Paretian model relative to the Gaussian model shows that in reality extreme events have larger probability than predicted by the Gaussian distribution, especially in the extreme momentum deciles. The obtained results suggest that the Gaussian hypothesis for the postulated model of the momentum decile returns is inadequate. The results imply that the stable Paretian hypothesis better explains the tails and the central part of the return distribution.

3.4 Performance Evaluation of Risk-Return Ratio Criteria Based on Independent Performance Measure

In addition to evaluating the performance of the ratio criteria of the momentum strategy with regards to total cumulative realized return (accumulated profits or winner-loser spreads) over the observed period, we also apply an independent performance measure [Insert Footnote 10 here] to evaluate ratios on risk-adjusted performance based on an appropriate measure of risk in a uniform manner. To that purpose, we introduce a risk-adjusted independent performance measure, $E(X_t)/CVaR_{99\%}(X_t)$, in the form of the $STARR_{99\%}$ ratio, where X_t is the sequence of the daily spreads (difference between winner and loser portfolio returns over the observed period) for which we calculate the expected return in the numerator and the expected shortfall in the denominator. The ratio criterion that obtains the best risk-adjusted performance is the one that attains the highest value of the independent performance measure.

The results of the evaluation of the reward-risk criteria on cumulative return, Sharpe ratio and independent performance measure are shown in Table 5. For our 6-month/6-month strategy, the best performance value of 0.0218 for independent performance measure is obtained for the R-ratio(99%,99%) with a cumulative profit of 2.7919. It is obvious that evaluation of performance using total cumulative return (final wealth) and risk-adjusted measures may provide different performance rankings. For example, while the cumulative return and STARR(95%) ratio obtain the top performance on cumulative return measure, their risk-adjusted performance measured on the independent performance measure and the Sharpe ratio is average. In contrast, the R-ratio(99%,99%) and the R-ratio(91%,91%) obtain the best risk-adjusted performance on the independent performance measure, but obtain the seventh and third best cumulative spread respectively. The relative rankings of criteria based on the Sharpe ratio and independent perfor mance measure coincide.

Finally, we analyze the ranking performance of cumulative return criterion and the Sharpe ratio criterion based on the independent performance measure for our 6-month/6-month strategy. Cumulative return criterion obtains fourth ranking position and the Sharpe ratio criterion obtains one of the lowest performance ranking (ninth ranking position) measured by the independent performance measure. This implies that the Sharpe ratio as the risk-adjusted criterion for a given set of data is not providing an optimal risk-adjusted performance. Cumulative return criterion, although choosing the winners with the highest return, seems to choose among them a high proportion of winners that are also persistent in risk-adjusted performance. The R-ratio(99%, 99%) and the R-ratio(91%,91%) are the two best performing ratio criteria based on the independent performance measure for our strategy. To summarize, although the cumulative return criterion obtains the highest cumulative profits, the alternative R-ratios obtain the best risk-adjusted performance based on the independent performance measure.

[Insert Table 5 here]

5. Conclusions

In this study, we apply the alternative reward-risk criteria to momentum portfolio construction. The alternative ratio criteria that account for a risk-return profile of the individual stocks are based on the coherent risk measure of the expected shortfall and are not restricted to the normal return distribution assumption. They can be conveniently applied at the individual stock and portfolio level. Additionally, reward-risk ratio criteria values are computed using daily data which enable better capture of the distributional properties of stock returns and their risk component at a different threshold level of the tail distribution. Alternative ratios drive balanced risk-return performance according to captured risk-return profiles of observed stocks in a sample. For examined 6-month/6-month strategy, although the cumulative return criterion provides the highest realized annualized return of 15.36%, the alternative R-ratio provides still high annualized return of 10.32% and the better risk-adjusted performance than a simple cumulative return and the traditional Sharpe ratio criterion.

We perform distributional analysis of the momentum deciles within framework of general stable distributions and find that extreme winner and loser decile portfolios have unique characteristics with regards to stable parameter estimates of their returns. We observe a systematic pattern of index of stability for extreme winner and loser deciles that have lower tail index that of middle deciles. It is not surprising that those assets should have also the highest tail-volatility of the return distributions. This suggests that winner and loser portfolio returns imply significant component due to heavy tails when compared to other deciles. In addition, we also reveal a systematic skewness pattern for winner and loser deciles, in that loser deciles have higher skewness than that of the winner portfolio for every criteria. The results suggest that achieving momentum profits implies the acceptance of negative skewness. For investor that considers only the cumulative return for the momentum strategy, he will have to accept higher tail-risk whereas the investor that pursue strategies based on reward-risk criteria will obtain the best risk-return profile with lower tail risk.

In order to evaluate risk-adjusted performance, we evaluate and compare different reward-risk ratios and cumulative return benchmark using an independent performance measure in the form of $STARR_{99\%}$ ratio. Our results confirm that the alternative R-ratio and the STARR ratios capture the features of the data and their distributional behavior considerably better than the classical mean-variance model embodied by the Sharpe ratio which underperforms on cumulative spread and independent performance measure. The reason behind this good performance of the alternative ratios lies in the compliance with the coherent risk measure's ability to capture distributional features of data including the component of risk due to heavy tails, and the property of parameters to adjust for the upside reward and downside risk simultaneously.

The future research shall explore the risk-based explanations based on the market model and macroeconomic multifactor models and link the observations to observed distributional properties of momentum portfolios.

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Footnotes

1. The observation by Mandelbrot (1963) and Fama (1963, 1965) of excess kurtosis in empirical financial return processes led them to reject the normal assumption and propose non-Gaussian stable processes as a statistical model for asset returns. Non-Gaussian stable distributions are commonly referred to as "stable Paretian" distribution due to the fact that the tails of the non-Gaussian stable density have Pareto power-type decay.

2. In the mean-variance space, winners are positioned on the upper part of the efficient mean-variance frontier, so that by construction they entail the highest risk Losers simply correspond to the winners multiplied by -1. In the observed universe of assets bounded by mean-variance frontier, there may be riskier assets with higher variance (measured by standard deviation) than those of selected winners (or losers) but they will not qualify as winner or loser based on cumulative return criterion. Similar to return ranking criterion, we might think of applying a variance ranking criterion. However, the stocks with the greatest variance do not necessarily need to be the stocks with the greatest return at the same time and would not be on the efficient frontier.

3. This choice can be further complicated by considering which risk-adjusted criteria are more useful for an investor rather than another.

4. Expected shortfall conveys the information about the expected size of a loss exceeding VaR. For example, suppose that a portfolio's risk is calculated through the simulation. For 1,000 simulations and $a = 0.95$, the portfolio's VaR would be the smallest of the 50 largest losses. The corresponding expected shortfall would be then estimated by the numerical average of these 50 largest losses.

5. There are some notational inconsistencies in the current literature on expected shortfall. Some authors distinguish between expected tail loss and CVaR, so that only in the case of continuous random variable the definition of expected shortfall coincides with that of CVaR.

6. R-ratio stands for Rachev ratio and was introduced in the context of risk-reward alternative performance measures and risk estimation in portfolio theory (Biglova, Ortobelli, Rachev, and Stoyanov, 2004).

7. To obtain monthly returns, the daily returns are multiplied by 21 (assumption of 21 trading days per month)

8. The application of t-statistics requires the i.i.d. normal assumption for its (asymptotic) validity. However, it is well established in empirical finance that the iid normal assumption is systematically violated by asset returns in practice – returns typically exhibit skewness, excess kurtosis, (conditional) heteroscedasticity and temporal dependence. Limiting distributions for the t-statistics and the K-S-statistics are not parameter free due to the fact that parameters of the distribution have been estimated. Limiting distributions in this case can be obtained by bootstrapping, but this is beyond the scope of this paper. Here, we use the K-S-statistics only as a distance measure.

9. The ETL measures the expected value of portfolio returns given that the VaR has been exceeded. When the ETL concept is symmetrically applied to the appropriate part of the portfolio returns, a measure of portfolio reward is obtained (see Biglova, Ortobelli, Rachev and Stoyanov, 2004).

10. Unlike other studies which apply one-factor or multi-factor models as performance benchmark, we give preference to direct evaluation and comparison of performance on final wealth and risk-adjusted performance measure using daily data. The portfolio alpha's values from the factor models can be considerably influenced by the overly simplistic structure of the model with much of the idiosyncratic risk left unexplained.

Figure Captions

Figure 1. Estimate of skewness for momentum decile returns of a 6-month/6-month momentum strategy for cumulative return, Sharpe ratio and STARR(95%) criterion

Figure 2: Cumulative realized momentum profits for different reward-risk ratio criteria and 6-month/6-month strategy.

Figure 3: Estimate of the tail index for momentum decile returns of a 6-month/6-month momentum strategy for cumulative return, Sharpe ratio and R-ratio(91%,91%)

Table 1. Momentum Portfolio Returns and Summary Statistics

Reward-Risk Ratio	Portfolio	Descriptive Statistics			
		Mean	**Volatility**	**Skewness**	**Kurtosis**
Cumulative return (Benchmark)	Winner	0.0300	0.0187	-0.0163	2.9218
	Loser	0.0172	0.0170	0.48149	5.5751
	Winner-Loser	0.0128	-	-	-
Sharpe Ratio	Winner	0.0231	0.0152	0.01770	4.6177
	Loser	0.0164	0.0148	0.5146	5.9533
	Winner - Loser	0.0067	-	-	-
STARR (99%)	Winner	0.0226	0.0148	-0.0134	3.8088
	Loser	0.0165	0.0155	0.57800	5.5742
	Winner-Loser	0.0060	-	-	-
STARR (95%)	Winner	0.0245	0.0152	-0.0206	4.2520
	Loser	0.0165	0.0152	0.5300	5.2106
	Winner-Loser	0.0079	-	-	-
STARR (90%)	Winner	0.0235	0.0152	-0.0096	4.0685
	Loser	0.0165	0.0151	0.5095	5.7897
	Winner-Loser	0.0069	-	-	-
STARR (75%)	Winner	0.0235	0.0150	0.0030	4.1790
	Loser	0.0159	0.0148	0.5085	5.9717
	Winner-Loser	0.0075	-	-	-
STARR (50%)	Winner	0.0235	0.0149	0.0023	4.5457
	Loser	0.0163	0.0146	0.5153	6.1469
	Winner-Loser	0.0071	-	-	-
R-ratio (99%,99%)	Winner	0.0214	0.0131	-0.0285	2.6163
	Loser	0.0138	0.0116	-0.0073	2.3276
	Winner-Loser	0.0075	-	-	-
R-ratio (95%,95%)	Winner	0.0214	0.0134	-0.06168	2.9597
	Loser	0.0147	0.0118	0.02686	3.1761
	Winner-Loser	0.0067	-	-	-

Table 1. Cont.

Reward-Risk Ratio	Portfolio	Mean	Volatility	Skewness	Kurtosis
R-ratio (91%,91%)	Winner	0.0226	0.0135	-0.1261	3.1018
	Loser	0.0140	0.0117	0.0375	3.3995
	Winner-Loser	0.0086	-	-	-
R-ratio (50%,99%)	Winner	0.0191	0.0144	0.0613	2.7349
	Loser	0.0159	0.0123	0.0222	3.7835
	Winner-Loser	0.0031	-	-	-
R-ratio (50%,95%)	Winner	0.0201	0.0145	-0.0002	2.8884
	Loser	0.0180	0.0122	0.0126	4.6842
	Winner-Loser	0.0021	-	-	-

The equally weighted portfolios of S&P 500 equities using 6/6 momentum strategy for different ranking criteria are formed. For each 6 month momentum, we rank the sample of 500 stocks in the S&P universe over the period 1996 – 2003 into 10 equally weighted portfolios and follow the post-portfolio formation return over holding period of 6 months. The table reports average monthly return, volatility, skewness and kurtosis for the loser portfolio (first decile) and winner portfolio (tenth decile). Monthly returns are aggregated from daily returns assuming 21 trading days in month. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest values of the stock selection criteria over the past 6-months, and winner (P10) comprises the stocks with the highest values of the stock selection criteria over the past 6-months.

Table 2. Final Wealth of Momentum Portfolios

Reward-Risk Ratio	Portfolio	Final Wealth
Cumulative return (Benchmark)	Loser	3.5658
	Winner	10.5592
	Winner-Loser	6.9934
Sharpe Ratio	Loser	3.6141
	Winner	6.4059
	Winner - Loser	2.7917
STARR (99%)	Loser	3.5306
	Winner	6.1856
	Winner-Loser	2.6550
STARR (95%)	Loser	3.6038
	Winner	7.2652
	Winner-Loser	3.6614
STARR (90%)	Loser	3.5744
	Winner	6.6241
	Winner-Loser	3.0497
STARR (75%)	Loser	3.4283
	Winner	6.7135
	Winner-Loser	3.2852
STARR (50%)	Loser	3.5729
	Winner	6.8325
	Winner-Loser	3.2596
R-ratio (99%,99%)	Loser	3.0539
	Winner	5.844946
	Winner-Loser	2.7913
R-ratio (95%,95%)	Loser	3.3021
	Winner	5.7529
	Winner-Loser	2.4508
R-ratio (91%,91%)	Loser	3.1119
	Winner	6.4995
	Winner-Loser	3.3875
R-ratio (50%,99%)	Loser	3.6155
	Winner	4.5559
	Winner-Loser	0.9404

R-ratio (50%,95%)	Loser	4.4227
	Winner	5.0091
	Winner-Loser	0.5864

This table reports the final wealth for momentum portfolios at the end of observed period for different reward-risk selection criteria. The final wealth of winner and loser portfolios is the total realized return on these portfolios at the end of observed period respectively. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest values of criteria over the past *6*-months, and winner (P10) comprises the stocks with the highest values of criteria over the past *6*-months. Annualized returns for the specific strategy and risk-return ratio are given in parentheses. The sample includes a total of 500 stocks during the period of January 1996 to December 2003.

Table 3. Momentum Portfolio Returns and Estimates of Parameters of Stable Distribution

Reward-Risk Ratio	Parameter Estimate	Momentum Deciles									
		1 (Losers)	2	3	4	5	6	7	8	9	10 (Winn-ers)
Cumulative return (Benchmark)	m	0.00082	0.00063	0.00054	0.00061	0.00067	0.00064	0.00073	0.00070	0.00089	0.00143
	a	1.56704	1.63614	1.71760	1.72691	1.79380	1.80571	1.79596	1.83524	1.8683	1.7500
	b	0.03715	-0.0133	-0.0118	-0.0809	0.00820	-0.07826	-0.08899	-0.03086	0.0944	-0.0704
	s	0.00878	0.00715	0.00682	0.00639	0.00675	0.00664	0.00693	0.00720	0.0084	0.0111
Sharpe Ratio	m	0.00079	0.00066	0.00061	0.00069	0.00065	0.00082	0.00068	0.00080	0.00083	0.00110
	a	1.5952	1.6733	1.7555	1.7317	1,7769	1,8471	1,8419	1,8407	1,8054	1,6584
	b	0,0086	-0.0046	-0,022	-0,0649	0,0774	-0,1216	0,0613	-0,0201	-0,0182	-0,0946
	s	0.0077	0,0076	0,0074	0,0071	0,0073	0,0072	0,0076	0,0073	0,0078	0,0083
STARR (99%)	m	0.00079	0.00064	0.00061	0.00062	0.00084	0.00074	0.00067	0.00073	0.00087	0.00108
	a	1.5786	1.6600	1.7256	1.7615	1.7949	1.8470	1.8562	1.8263	1.8257	1.7063
	b	0.0284	-0.0276	-0.0438	-0.0702	0.0498	-0.0817	-0.0536	0.0484	0.0120	-0.0816
	s	0.0080	0.0074	0.0073	0.0070	0.0073	0.0073	0.0076	0.0073	0.0080	0.0084
STARR (95%)	m	0.00079	0.00065	0.00062	0.00060	0.00075	0.00081	0.00068	0.00080	0.00078	0.00117
	a	1.5868	1.6648	1.7438	1.7428	1.7895	1.8360	1.8498	1.8564	1.7861	1.6855
	b	-0.0055	0.0100	-0.0572	-0.0576	0.0397	-0.0073	-0.0432	0.0043	0.0277	-0.0689
	s	0.0079	0,0075	0,0073	0,0069	0,0073	0,0072	0,0075	0,0075	0,0077	0,0085
STARR (90%)	m	0.00079	0.00069	0.00059	0.00065	0.00067	0.00085	0.00067	0.00080	0.00082	0.00112
	a	1.5838	1.6793	1.7574	1.7452	1.7905	1.8532	1.8532	1.8516	1.7973	1.6692
	b	-0.0044	0.0169	-0.0183	-0.0478	0.0721	-0.0778	-0.0107	-0.0188	-0.0071	-0.0740
	s	0.0078	0.0076	0.0074	0.0069	0.0074	0.0072	0.0076	0.0074	0.0077	0.0084
STARR (75%)	m	0.00076	0.00068	0.00061	0.00063	0.00070	0.00080	0.00069	0.00078	0.00086	0.00112
	a	1.5915	1.6600	1.7509	1.7621	1.7871	1.8445	1.8382	1.8388	1.8118	1.6650
	b	-0.0010	-0.0079	-0.0068	-0.0279	0.0681	-0.0789	-0.0041	-0.0129	-0.0061	-0.0846
	s	0.0077	0.0077	0.0073	0.0072	0.0074	0.0072	0.0076	0.0074	0.0079	0.0082
STARR (50%)	m	0.00078	0.00068	0.00061	0.00064	0.00068	0.00081	0.00068	0.00078	0.00085	0.00113
	a	1.5969	1.6611	1.7554	1.7410	1.7846	1.8408	1.8445	1.8388	1.8125	1.6539
	b	0.0047	-0.0123	0.0031	-0.0424	0.0970	-0.1204	0.0362	-0.0164	-0.0192	-0.0962
	s	0.0076	0.0077	0.0073	0.0072	0.0074	0.0072	0.0076	0.0074	0.0079	0.0081

Table 3. Cont.

Reward-Risk Ratio	Parameter Estimate	Momentum Deciles									
		1 (losers)	2	3	4	5	6	7	8	9	10 (winners)
R-ratio (99%,99%)	*m*	0.00066	0.00067	0.00066	0.00068	0.00075	0.00076	0.00084	0.00081	0.00080	0.00102
	a	1.7727	1.7526	1.7927	1.8015	1.7909	1.8082	1.8152	1.8028	1.7285	1.7779
	b	-0.0657	-0.1155	-0.0659	-0.1098	0.0011	0.0375	0.1399	0.0722	0.0027	0.0352
	s	0.0071	0.0072	0.0070	0.0071	0.0073	0.0074	0.0079	0.0077	0.0076	0.0080
R-ratio (95%,95%)	*m*	0.00070	0.00055	0.00068	0.00072	0.00075	0.00073	0.00077	0.00077	0.00089	0.00102
	a	1.7526	1.7539	1.8136	1.7333	1.8190	1.8073	1.7754	1.8325	1.7624	1.7811
	b	-0.0733	-0.0690	-0.0497	-0.1063	0.0125	0.0908	0.0837	0.1161	0.0028	-0.0438
	s	0.0070	0.0069	0.0072	0.0070	0.0074	0.0075	0.0079	0.0075	0.0080	0.0081
R-ratio (91%,91%)	*m*	0.00067	0.00061	0.00070	0.00071	0.00070	0.00069	0.00077	0.00086	0.00083	0.00108
	a	1.7196	1.7652	1.7981	1.7679	1.7867	1.7894	1.8202	1.7799	1.8128	1.7858
	β	-0.0487	-0.0940	-0.1246	-0.0488	0.0182	0.0580	0.1854	0.0705	0.0325	-0.0803
	σ	0.0068	0.0072	0.0072	0.0071	0.0072	0.0073	0.0079	0.0078	0.0082	0.0082
R-ratio (50%,99%)	*m*	0.00076	0.00071	0.00078	0.00077	0.00070	0.00074	0.00079	0.00063	0.00083	0.00091
	α	1.6911	1.7994	1.7097	1.8352	1.7553	1.7889	1.7862	1.8147	1.7990	1.7978
	β	-0.0688	-0.0688	-0.1138	0.0186	-0.1043	0.0621	0.0053	0.0682	0.0523	0.0716
	σ	0.0070	0.0071	0.0072	0.0072	0.0072	0.0075	0.0073	0.0076	0.0079	0.0088
R-ratio (50%,95%)	*m*	0.00086	0.00073	0.00067	0.00070	0.00068	0.00071	0.00074	0.00081	0.00078	0.00096
	α	1.6943	1.7218	1.7352	1.7714	1.7694	1.8071	1.8376	1.8013	1.7956	1.7865
	β	-0.1189	-0.0815	-0.0397	-0.0780	-0.0315	0.0344	0.1321	-0.0334	-0.0196	0.0408
	σ	0.0068	0.0071	0.0070	0.0072	0.0074	0.0075	0.0075	0.0075	0.0084	0.0088

The equally weighted portfolios of S&P 500 equities using 6/6 momentum strategy for different ranking criteria are formed. For each 6 month momentum, we rank the sample of stocks in the S&P universe over the period 1996 – 2003 into 10 equally weighted portfolios and follow the post-portfolio formation return over holding period of 6 months. The table reports the location parameter *m*, index of stability *a*, skewness parameter *b* and the scaling parameter *s* for the momentum deciles. For *a* > 1, as is the case here, the location parameter *m* is the usual mean estimator. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest value of the criteria over the past 6-months, and winner (P10) comprises the stocks with the highest value of the criteria over the past 6-months.

Table 4. Goodness-of-fit Statistics for Distribution of Momentum Portfolio Returns

Reward-Risk Ratio	Goodness-of-Fit Statistic	Portfolio									
		1 (Losers)	2	3	4	5	6	7	8	9	10 (Winn-ers)
Cumulative return (Benchmark)	KS normal	0.0774	0.0745	0.0572	0.0588	0.0514	0.0414	0.0570	0.0377	0.0370	0.0467
	KS stable	0.0160	0.0297	0.0210	0.0182	0.0194	0.0183	0.0274	0.0168	0.0181	0.0226
	AD normal	0.2548	0.4353	0.5726	0.3607	0.3607	0.4137	0.3078	0.3607	0.3078	0.4137
	AD stable	0.0798	0.0857	0.0715	0.0604	0.0455	0.0458	0.0572	0.0458	0.0520	0.0510
Sharpe Ratio	KS normal	0.0852	0.0619	0.0578	0.0488	0.0561	0.0425	0.0493	0.0356	0.0492	0.0695
	KS stable	0.0177	0.0256	0.0221	0.0267	0.0252	0.0160	0.0232	0.0160	0.0258	0.0204
	AD normal	0.3607	0.3607	0.6785	0.3773	0.4137	0.3607	0.3078	0.2105	0.3607	0.5196
	AD stable	0.0695	0.0715	0.0600	0.0678	0.0584	0.0493	0.0526	0.0476	0.0601	0.0584
STARR (99%)	KS normal	0.0708	0.0596	0.0536	0.0526	0.0422	0.0482	0.0379	0.0530	0.0443	0.0556
	KS stable	0.0210	0.0176	0.0223	0.0273	0.0160	0.0226	0,0170	0.0312	0.0190	0.0259
	AD normal	0.4137	0.3607	0.4137	0.4667	0.2579	0.3607	0.4137	0.2548	0.3607	0.5102
	AD stable	0.0908	0.0633	0.0684	0.0614	0.0507	0.0499	0.0505	0.0742	0.0574	0.0540
STARR (95%)	KS normal	0.0754	0.0596	0.0552	0.0590	0.0520	0.0411	0.0350	0.0339	0.0500	0.0605
	KS stable	0.0176	0.0249	0.0267	0.0264	0.0240	0.0225	0.0198	0.0211	0.0259	0.0145
	AD normal	0.3607	0.4137	0.4727	0.2579	0.3607	0.3607	0.4137	0.3078	0.4667	0.3607
	AD stable	0.0756	0.0609	0.0592	0.0641	0.0549	0.0496	0.0436	0.0564	0.0563	0.0542
STARR (90%)	KS normal	0.0807	0.0610	0.0468	0.0472	0.0475	0.0335	0.0451	0.0399	0.0409	0.0680
	KS stable	0.0200	0.0249	0.0249	0.0209	0.0261	0.0171	0.0238	0.0257	0.0224	0.0167
	AD normal	0.3607	0.4667	0.6785	0.6785	0.3607	0.4137	0.2548	0.2548	0.2548	0.5196
	AD stable	0.0722	0.0726	0.0712	0.0549	0.0555	0.0481	0.0536	0.0553	0.0589	0.0579
STARR (75%)	KS normal	0.0767	0.0623	0.0509	0.0571	0.0647	0.0399	0.0413	0.0403	0.0424	0.0681
	KS stable	0.0169	0.0203	0.0192	0.0294	0.0304	0.0179	0.0304	0.0196	0.0212	0.0239
	AD normal	0.4667	0.4137	0.6256	0.6256	0.4667	0.4137	0.4137	0.3078	0.2480	0.4667
	AD stable	0.0746	0.0826	0.0608	0.0651	0.0631	0.0384	0.0658	0.0480	0.0502	0.0520
STARR (50%)	KS normal	0.0711	0.0639	0.049	0.0548	0.0561	0.0390	0.0422	0.0368	0.0496	0.0646
	KS stable	0.0201	0.0255	0.0237	0.0334	0.0229	0.0179	0.0198	0.0234	0.0305	0.0116
	AD normal	0.5726	0.4137	0.4667	0.3607	0.2783	0.3607	0.3607	0.2480	0.2548	0.4667
	AD stable	0.0757	0.0743	0.0495	0.0672	0.0706	0.0440	0.0463	0.0500	0.0664	0.0617

Table 4. Cont.

Reward-Risk Ratio	Goodness-of-Fit Statistic	Portfolio									
		1 (Losers)	2	3	4	5	6	7	8	9	10 (Winn-ers)
R-ratio (99%,99%)	KS normal	0.0536	0.0513	0.0485	0.0404	0.0565	0.0508	0.0478	0.0450	0.0572	0.0454
	KS stable	0.0189	0.0266	0.0307	0.0245	0.0270	0.0247	0.0244	0.0254	0.0271	0.0223
	AD normal	0.3078	0.4137	0.3607	0.3078	0.3078	0.2480	0.3607	0.3078	0.2566	0.3229
	AD stable	0.0512	0.0609	0.0638	0.0539	0.0588	0.0560	0.0520	0.0601	0.0616	0.0633
R-ratio (95%,95%)	KS normal	0.0543	0.0523	0.0426	0.0491	0.0413	0.0535	0.0543	0.0565	0.0476	0.0503
	KS stable	0.0225	0.0200	0.0163	0.0194	0.0182	0.0202	0.0276	0.0260	0.0262	0.0219
	AD normal	0.3078	0.3607	0.2382	0.4667	0.3607	0.3078	0.2273	0.2548	0.3191	0.4137
	AD stable	0.0514	0.0577	0.0575	0.0684	0.0409	0.0522	0.0580	0.0579	0.0630	0.0528
R-ratio (91%,91%)	KS normal	0.0600	0.0520	0.0508	0.0500	0.0526	0.0528	0.0511	0.0526	0.0551	0.0496
	KS stable	0.0237	0.0168	0.0230	0.0194	0.0227	0.0223	0.0284	0.0324	0.0286	0.0224
	AD normal	0.4137	0.3607	0.3191	0.4137	0.3078	0.3607	0.3078	0.5196	0.3607	0.3607
	AD stable	0.0733	0.0553	0.0469	0.0500	0.0520	0.0616	0.0793	0.0658	0.0587	0.0504
R-ratio (50%,99%)	KS normal	0.0566	0.0392	0.0561	0.0424	0.0519	0.0614	0.0470	0.0520	0.0457	0.0506
	KS stable	0.0219	0.0202	0.0210	0.0173	0.0264	0.0263	0.0268	0.0322	0.0188	0.0256
	AD normal	0.3078	0.2548	0.4667	0.3229	0.5196	0.2579	0.2579	0.2548	0.4137	0.4137
	AD stable	0.0532	0.0661	0.0686	0.0614	0.0534	0.0599	0.0548	0.0668	0.0608	0.0584
R-ratio (50%,95%)	KS normal	0.0627	0.0522	0.0553	0.0462	0.0504	0.0511	0.0473	0.0442	0.0560	0.0459
	KS stable	0.0266	0.0261	0.0163	0.0237	0.0383	0.0207	0.0190	0.0218	0.0293	0.0205
	AD normal	0.3191	0.6785	0.4137	0.4137	0.3607	0.3078	0.3078	0.2548	0.5196	0.6785
	AD stable	0.0582	0.0677	0.0625	0.0584	0.0770	0.0483	0.0615	0.0538	0.0591	0.0706

The equally weighted portfolios of S&P 500 equities using 6/6 momentum strategy for different ranking criteria are formed. For each 6 month momentum, we rank the sample of 500 stocks in the S&P universe over the period 1996 – 2003 into 10 equally weighted portfolios and follow the post-portfolio formation return over holding period of 6 months. The table reports the Kolmogorov distance (KD) and the Anderson-Darling (AD) statistic assuming the unconditional homoscedastic distributional model of i.i.d. decile returns. Loser (P1) is the equally weighted portfolio of 10% of the stocks with the lowest value of criteria over the past 6-months, and winner (P10) comprises the stocks with the highest value of criteria over the past 6-months.

Table 5. Performance Evaluation of Momentum Spreads generated by Reward-Risk Ratio Strategies using Cumulative Spread, Sharpe ratio and Independent Performance Measure

Reward-Risk Ratio	Cumulative spread	Sharpe Ratio	$E(X_t)/CVaR_{99\%}(X_t)$
Cumulative Return	**6.9934**	0.035662	0.01398
Sharpe Ratio	2.7917	0.020984	0.008033
STARR (99%)	2.6550	0.019599	0.007587
STARR (95%)	3.6614	0.025233	0.009786
STARR (90%)	3.04972	0.022436	0.008582
STARR (75%)	3.28522	0.024768	0.009489
STARR (50%)	3.2596	0.024137	0.009277
R-ratio (99%,99%)	2.7919	**0.049808**	**0.021775**
R-ratio(95%,95%)	2.45081	0.036602	0.015508
R-ratio(91%,91%)	3.38759	0.045162	0.018721
R-ratio(50%,99%)	0.94043	0.015574	0.0066
R-ratio(50%,95%)	0.59642	0.008904	0.00366

This table reports the evaluation of momentum strategies using risk-adjusted criteria and cumulative return benchmark criterion on cumulative spread, Sharpe ratio, and independent performance measure. The values in bold denote the best criterion performance for specific evaluation measure. Independent performance measure is risk-adjusted performance measure in the form of $STARR_{99\%}$ ratio. The sample includes a total of 582 stocks in the S&P universe during the period of January 1996 to December 2003.

Figure 1

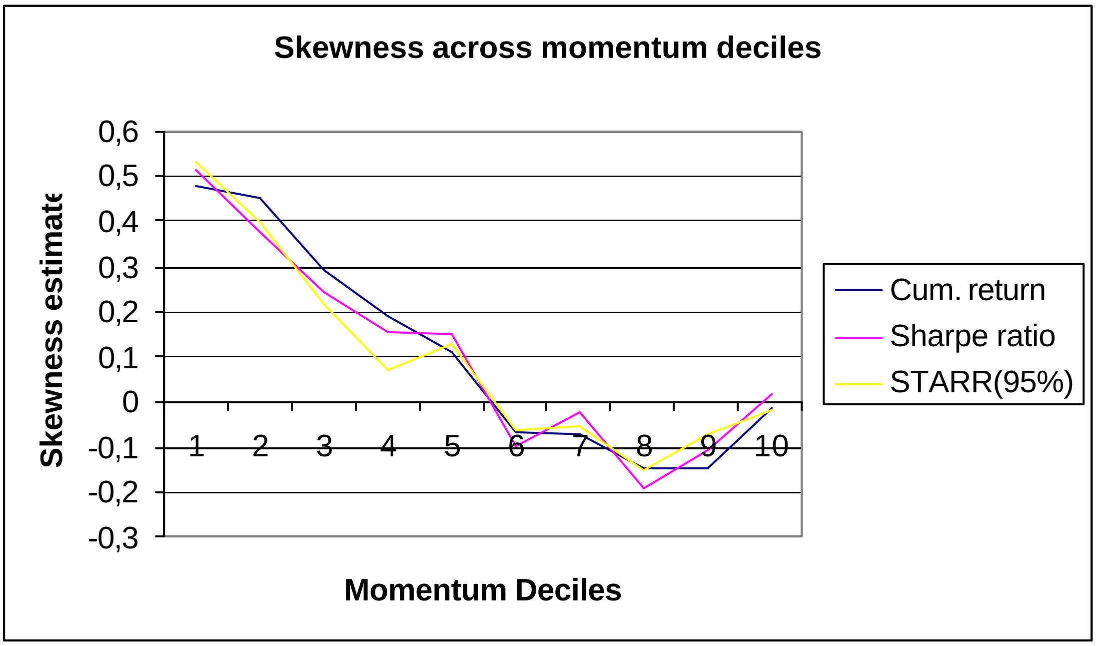
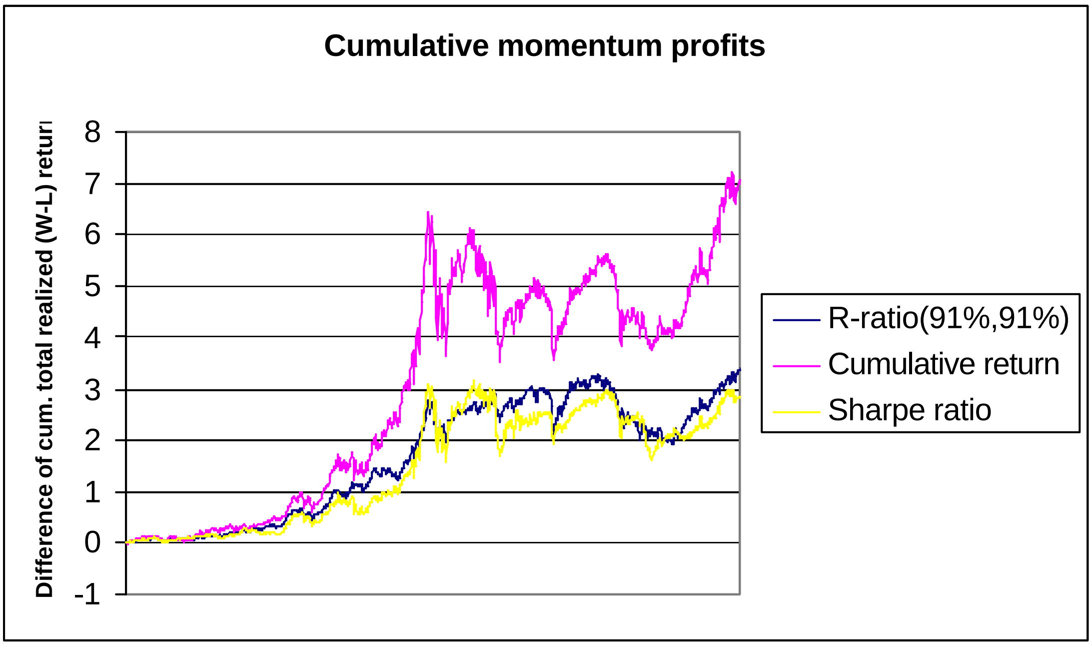
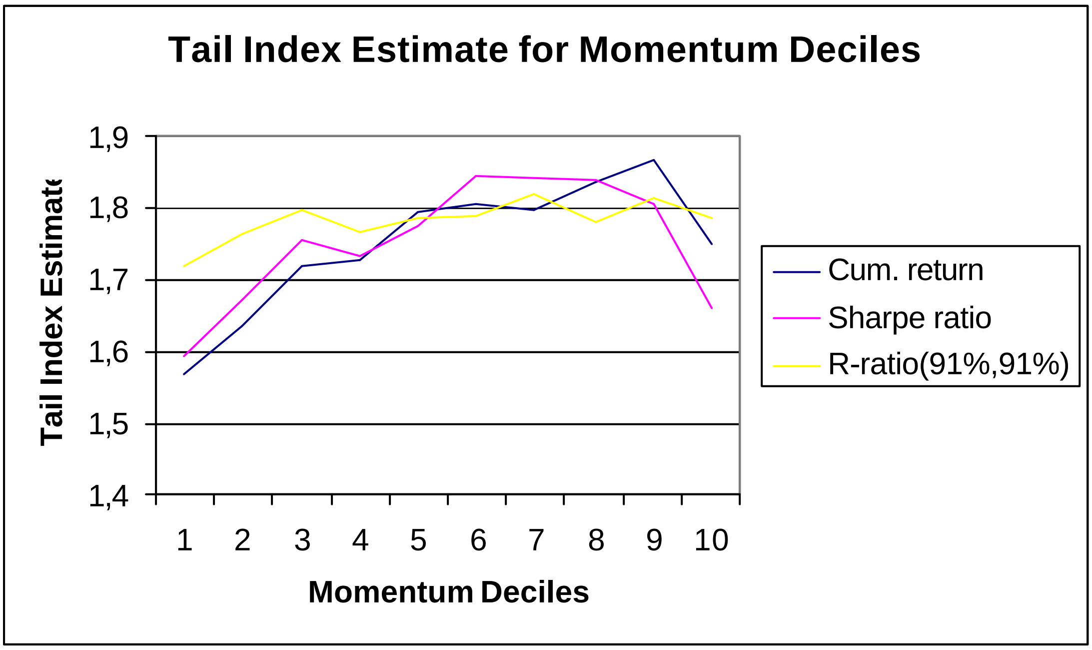

Skewness across momentum deciles
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
Skewness estimate
1
2
3
4
5
6
7
8
9
10
Momentum Deciles
Cum. return
Sharpe ratio
STARR(95%)

Figure 2



Cumulative momentum profits
Difference of cum. total realized (W-L) returi
8
7
6
5
4
3
2
1
0
-1
R-ratio(91%,91%)
Cumulative return
Sharpe ratio

Figure 3


Tail Index Estimate for Momentum Deciles
1,9
1,8
1,7
1,6
1,5
1,4
1 2 3 4 5 6 7 8 9 10
Momentum Deciles
Cum. return
Sharpe ratio
R-ratio(91%,91%)