On uniqueness for stochastic heat equations with non-Lipschitz coefficients

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We consider the question of uniqueness of solution to stochastic partial differential equations (SPDEs). We focus on the case of a particular parabolic SPDE — the heat equation perturbed by a multiplicative noise, or the stochastic heat equation.

Two important types of uniqueness are discussed: pathwise uniqueness and uniqueness in probability law of the solution. Under Lipschitz assumptions on noise coefficients, the pathwise uniqueness for a large class of SPDEs has been known for a long time. For non-Lipschitz SPDEs, uniqueness in law has been known in some very specific cases.

We describe results of joint work with Edwin Perkins and Anja Sturm on pathwise uniqueness for the stochastic heat equation

$$\frac{\partial}{\partial t} u(t, x) = \frac{1}{2} \Delta u(t, x) dt + \sigma(u(t, x)) \dot{W}(x, t), \quad t \geq 0, \ x \in \mathbb{R}^d$$

driven by Gaussian noise $\dot{W}$ on $\mathbb{R}^d \times \mathbb{R}_+$. $\dot{W}$ is white in time and “colored” in space. The case of non-Lipschitz coefficients $\sigma$ and singular spatial noise correlations is considered.