# Metamodels and the valuation of large variable annuity portfolios

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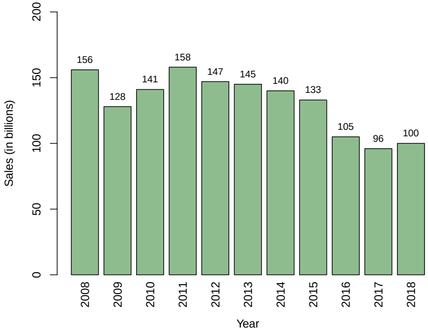
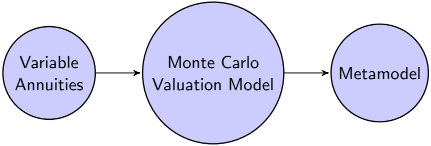
First Annual UCSB InsurTech Summit
University of California, Santa Barbara
Friday, May 3, 2019

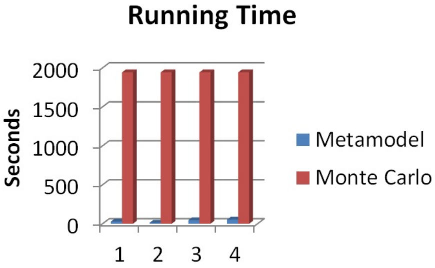
# Efficient valuation of large variable annuity portfolios




1. A challenge



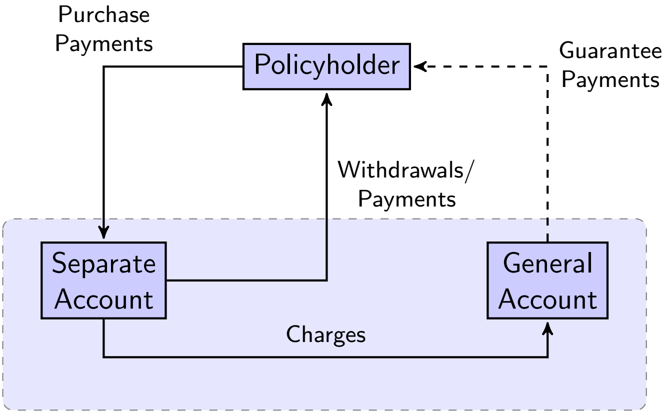

2. A metamodeling approach



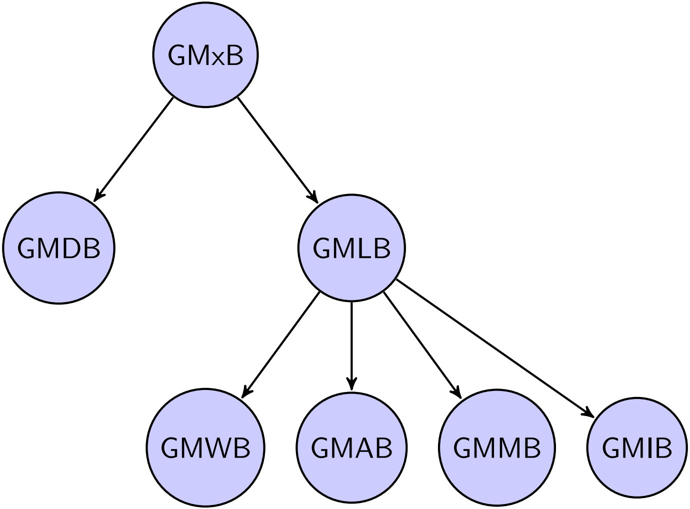

3. Numerical results

# What is a variable annuity?

A variable annuity is a retirement product, offered by an insurance company, that gives you the option to select from a variety of investment funds and then pays you retirement income, the amount of which will depend on the investment performance of funds you choose.



# Variable annuities come with guarantees



# Insurance companies have to make guarantee payments under bad market conditions

## Example (An immediate variable annuity with GMWB)

- Total investment and initial benefits base: \$100,000
- Maximum annual withdrawal: \$8,000

| Policy Year | INV Return | Fund Before WD | Annual WD | Fund After WD | Remaining Benefit | Guarantee CF |
|---|---|---|---|---|---|---|
| 1 | -10% | 90,000 | 8,000 | 82,000 | 92,000 | 0 |
| 2 | 10% | 90,200 | 8,000 | 82,200 | 84,000 | 0 |
| 3 | -30% | 57,540 | 8,000 | 49,540 | 76,000 | 0 |
| 4 | -30% | 34,678 | 8,000 | 26,678 | 68,000 | 0 |
| 5 | -10% | 24,010 | 8,000 | 16,010 | 60,000 | 0 |
| 6 | -10% | 14,409 | 8,000 | 6,409 | 52,000 | 0 |
| 7 | 10% | 7,050 | 8,000 | 0 | 44,000 | 950 |
| 8 | r | 0 | 8,000 | 0 | 36,000 | 8,000 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 12 | r | 0 | 8,000 | 0 | 4,000 | 8,000 |
| 13 | r | 0 | 4,000 | 0 | 0 | 4,000 |

# Dynamic hedging

Dynamic hedging is a popular approach to mitigate the financial risk, but

- Dynamic hedging requires calculating the dollar Deltas of a portfolio of variable annuity policies within a short time interval.
- The value of the guarantees cannot be determined by closed-form formula.
- The Monte Carlo simulation model is time-consuming.

There is also the additional computational issue related to reflect the effect of dynamic hedging in (quarterly) financial reporting.

# Use of Monte Carlo method

Using the Monte Carlo method to value large variable annuity portfolios is time-consuming:

Example (Valuing a portfolio of 100,000 policies)

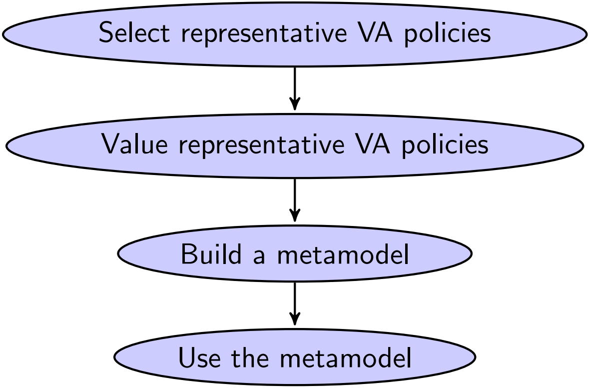
- 1,000 risk neutral scenarios
- 360 monthly time steps

$$\boxed{100,000 \times 1,000 \times 360 = 3.6 \times 10^{10}!}$$

$$\boxed{\frac{3.6 \times 10^{10} \text{ projections}}{200,000 \text{ projections/second}} = 50 \text{ hours!}}$$

# Metamodeling

- A metamodel, also a surrogate model, is a model of another model.
- Metamodeling has been applied to address the computational problems arising from valuation of variable annuity portfolios: a number of work published by co-author G. Gan.
- It involves four steps:



# Selecting representative policies

An important step in the metamodeling process is the selection of representative policies. Gan and Valdez (2016) compared five different experimental design methods for the GB2 regression model:

- Random sampling
- Low-discrepancy sequence
- Data clustering (hierarchical $k$-means)
- Latin hypercube sampling
- Conditional Latin hypercube sampling

# Some metamodels proposed/examined

We have studied and proposed some metamodels for the valuation of large VA portfolios:

- Ordinary kriging
- Universal kriging
- GB2 regression model
- Rank-order kriging (quantile kriging)
- Tree-based models - joint work with Z. Quan

Kriging has its origins in geostatistics or spatial analysis. It is in some sense an interpolation method that is closely related to the idea of regression.

# A portfolio of synthetic variable annuity policies

| Feature | Value |
|---|---|
| Policyholder birth date | [1/1/1950, 1/1/1980] |
| Issue date | [1/1/2000, 1/1/2014] |
| Valuation date | 1/1/2014 |
| Maturity | [15, 30] years |
| Account value | [50000, 500000] |
| Female percent | 40% |
| Product type | DBRP, DBRU, BBSU, etc. |
| Fund fee | 30, 50, 60, 80, 10, 38, 45, 55, 57, 46bps for Funds 1 to 10, respectively |
| Base fee | 200 bps |
| Rider fee | depends on product type |
| Number of funds invested | [1, 10] |

# VA product types in the synthetic portfolio

| Product | Description | Rider Fee |
|---|---|---|
| DBRP | GMDB with return of premium | 0.25% |
| DBRU | GMDB with annual roll-up | 0.35% |
| DBSU | GMDB with annual ratchet | 0.35% |
| ABRP | GMAB with return of premium | 0.50% |
| ABRU | GMAB with annual roll-up | 0.60% |
| ABSU | GMAB with annual ratchet | 0.60% |
| IBRP | GMIB with return of premium | 0.60% |
| IBRU | GMIB with annual roll-up | 0.70% |
| IBSU | GMIB with annual ratchet | 0.70% |
| MBRP | GMMB with return of premium | 0.50% |
| MBRU | GMMB with annual roll-up | 0.60% |
| MBSU | GMMB with annual ratchet | 0.60% |
| WBRP | GMWB with return of premium | 0.65% |
| WBRU | GMWB with annual roll-up | 0.75% |
| WBSU | GMWB with annual ratchet | 0.75% |
| DBAB | GMDB + GMAB with annual ratchet | 0.75% |
| DBIB | GMDB + GMIB with annual ratchet | 0.85% |
| DBMB | GMDB + GMMB with annual ratchet | 0.75% |
| DBWB | GMDB + GMWB with annual ratchet | 0.90% |

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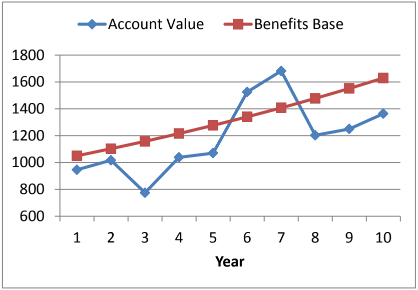
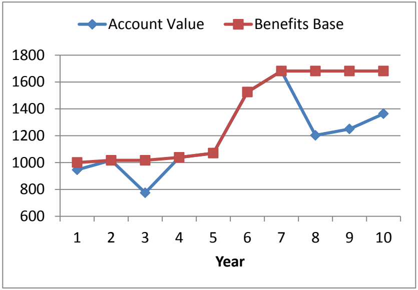
# VA provides guaranteed appreciation of the benefits base

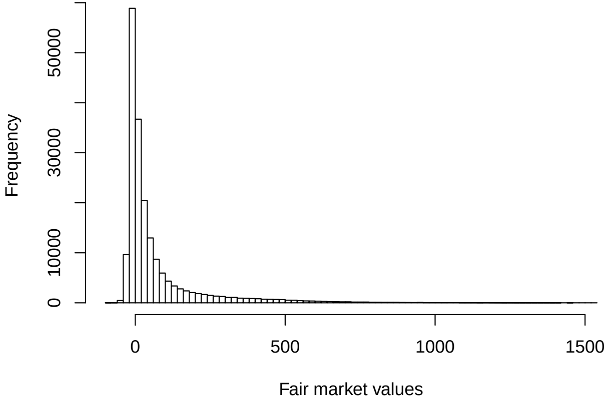



(Roll-up)




(Ratchet)

# Fair market values of the guarantees




| | Min. | 1st Qu. | Median | Mean | 3rd Qu. | Max. |
|---|---|---|---|---|---|---|
| fmv | -68.37 | -5.55 | 64.63 | 11.7 | 64.84 | 1210.32 |

# Training set - summary statistics - continuous variables

| Response variables | Description | Min. | 1st Q | Mean | Median | 3rd Q | Max. |
|---|---|---|---|---|---|---|---|
| gmwbBalance | GMWB balance | 0 | 0 | 27.8 | 0 | 0 | 422.26 |
| gbAmt | Guaranteed benefit amount | 51.88 | 183.98 | 323.29 | 306.89 | 437.36 | 920.62 |
| FundValue1 | Account value of the 1st fund | 0 | 0 | 32.02 | 12.62 | 46.76 | 629.89 |
| FundValue2 | Account value of the 2nd fund | 0 | 0 | 36.54 | 16.08 | 56.31 | 571.59 |
| FundValue3 | Account value of the 3rd fund | 0 | 0 | 26.78 | 11.81 | 36.64 | 458.78 |
| FundValue4 | Account value of the 4th fund | 0 | 0 | 25.8 | 10.48 | 38.29 | 539.36 |
| FundValue5 | Account value of the 5th fund | 0 | 0 | 22.29 | 10.54 | 34.71 | 425.92 |
| FundValue6 | Account value of the 6th fund | 0 | 0 | 37.15 | 19.64 | 53.96 | 654.64 |
| FundValue7 | Account value of the 7th fund | 0 | 0 | 28.78 | 12.88 | 42.56 | 546.89 |
| FundValue8 | Account value of the 8th fund | 0 | 0 | 31.27 | 15.59 | 46.24 | 529.57 |
| FundValue9 | Account value of the 9th fund | 0 | 0 | 31.93 | 13.9 | 45.17 | 599.44 |
| FundValue10 | Account value of the 10th fund | 0 | 0 | 32.6 | 13.86 | 45.09 | 510.43 |
| age | Age of the policyholder | 34.52 | 42.86 | 50.29 | 51.36 | 57.21 | 64.46 |
| ttm | Time to maturity in years | 0.75 | 10.09 | 14.61 | 14.6 | 19.12 | 27.52 |

# Tree-based models

Quan, Gan and Valdez (2019) compared the prediction performance of various tree-based models:

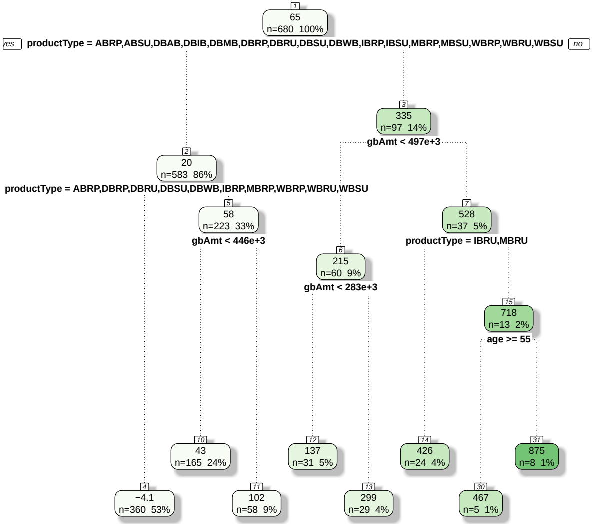
- Classification and Regression Trees (CART)
  - pruned by introducing penalty
- Ensemble methods: aggregate several regression trees to improve prediction accuracy
  - Bagging and random forests
  - Gradient boosting
- Unbiased recursive partitioning:
  - Conditional inference trees
  - Conditional random forests

# Unbiased recursive partitioning

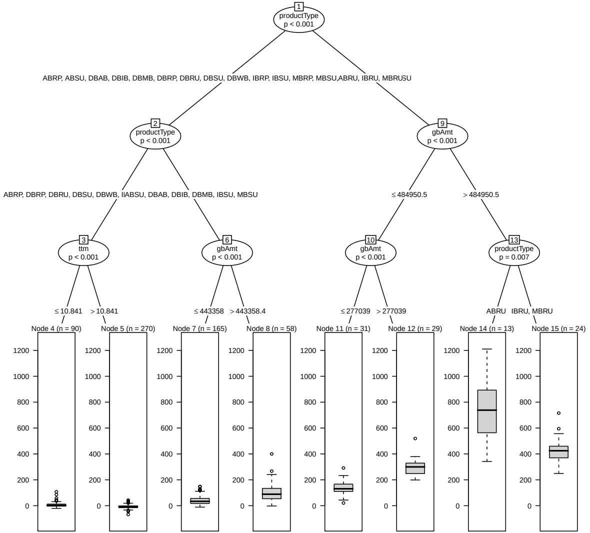
CART algorithms employ what is called recursive binary partitioning, which uses greedy search causing some drawbacks:

- Overfitting
  - Use a pruning process by applying cross-validation
- Bias in variable selection
  - Especially true when the explanatory variables present many possible splits or have missing values
  - Hothorn, et al. (2006) introduced conditional inference trees based on a partitioning of a statistic that is used to measure the association between the response and the explanatory variables.

# A regression tree



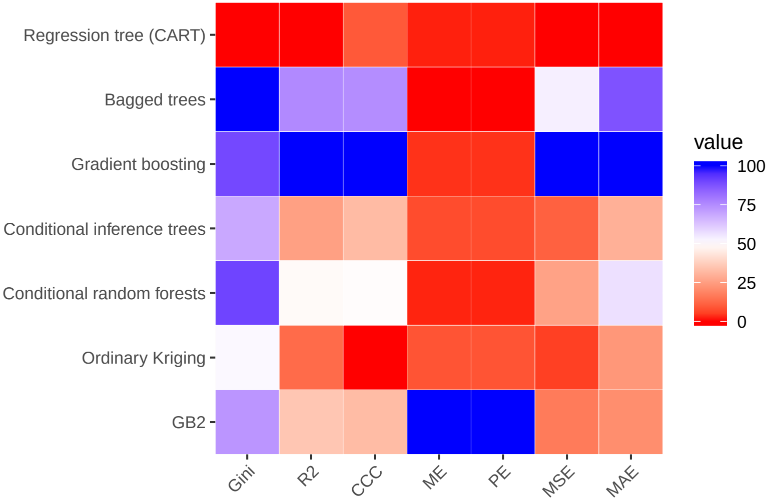
# A conditional inference tree



# Prediction accuracy of various models

| Model | $Gini$ | $R^2$ | $CCC$ | $ME$ | $PE$ | $MSE$ | $MAE$ |
|---|---|---|---|---|---|---|---|
| Regression tree (CART) | 0.786 | 0.845 | 0.917 | 1.678 | -0.025 | 3278.578 | 31.421 |
| Bagged trees | 0.842 | 0.918 | 0.954 | 2.213 | -0.033 | 1720.725 | 20.334 |
| Gradient boosting | 0.836 | 0.942 | 0.969 | 1.311 | -0.019 | 1214.899 | 19.341 |
| Conditional inference trees | 0.824 | 0.869 | 0.930 | 0.905 | -0.013 | 2754.853 | 26.536 |
| Conditional random forests | 0.836 | 0.892 | 0.940 | 1.596 | -0.024 | 2273.385 | 23.219 |
| Ordinary Kriging | 0.815 | 0.857 | 0.912 | -0.812 | 0.012 | 3006.192 | 27.429 |
| GB2 | 0.827 | 0.879 | 0.930 | 0.106 | -0.002 | 2554.246 | 27.772 |

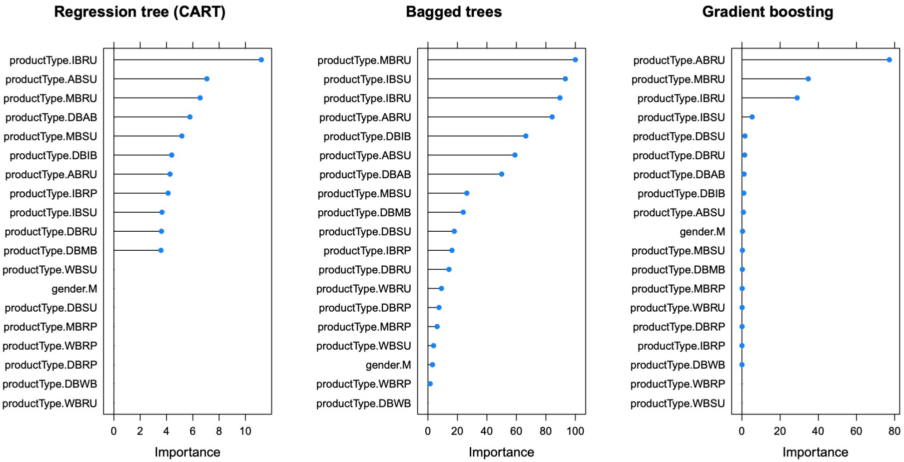
# A heatmap of model performance



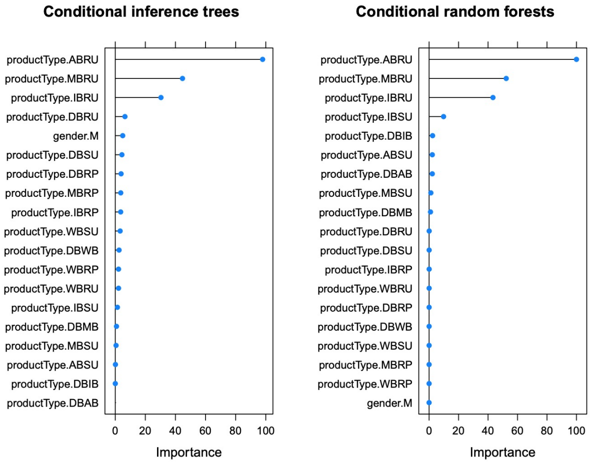
# Computational efficiency

| Model | Computation Time |
|---|---|
| Regression tree (CART) | 0.13 secs |
| Bagged trees | 2.70 secs |
| Gradient boosting | 4.69 secs |
| Conditional inference trees | 0.25 secs |
| Conditional random forests | 1214.72 secs |
| Ordinary Kriging | 277.49 secs |
| GB2 | 23.44 secs |

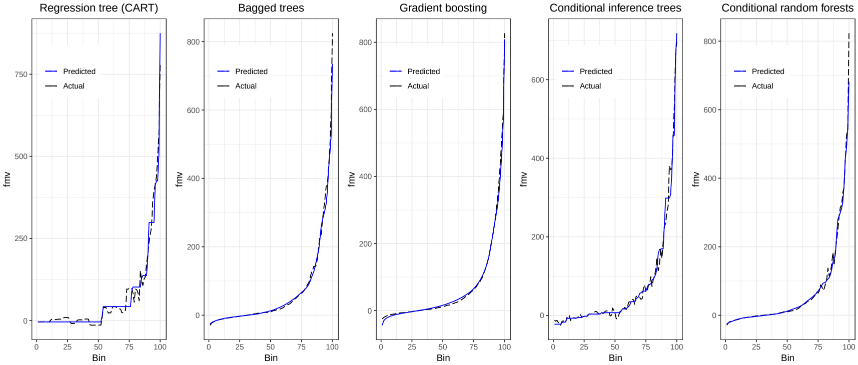
# Variable importance for tree-based models



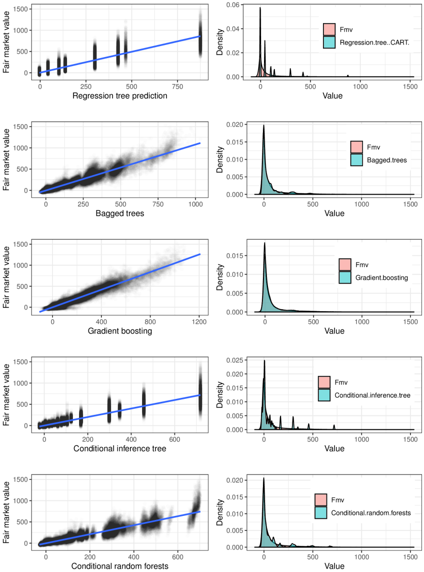
# Variable importance for tree-based models



# Lift curve plots - performance visualization



# Prediction and observed fair market values

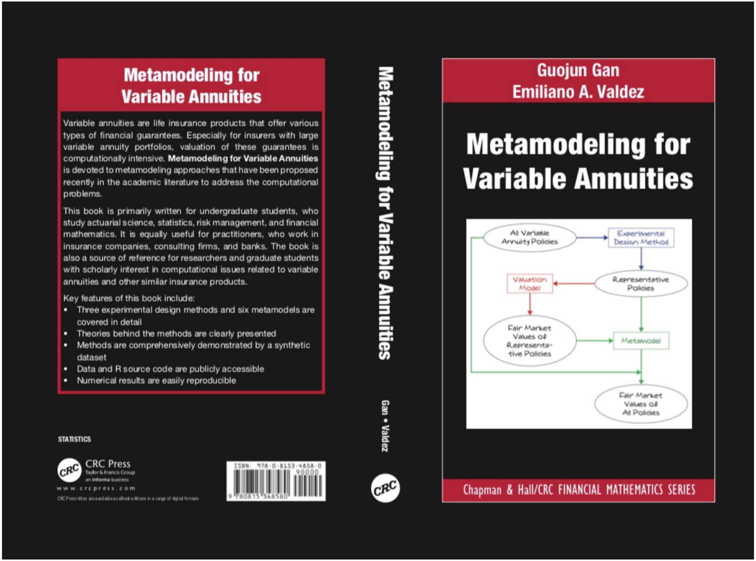


# Concluding remarks

We explore tree-based models and their extensions in developing metamodels for predicting fair market values. Besides computational efficiency and predictive accuracy, they have several advantages as an alternative predictive tool:

- Tree-based models are considered as nonparametric models that do not require distribution assumptions.
- Tree-based models can perform variable selection by assessing the relative importance.
- Tree-based models, especially with single smaller-sized trees, are straightforward to interpret by a visualization of the tree structure. This visualization was illustrated both in the case of regression tree and conditional inference tree.
- When compared to other metamodels for prediction purposes, tree-based models require less data preparation as they preserve the original scale to be more interpretable.

# Metamodeling book



# Appendix: Validation measures

| Validation measure | Description | Interpretation |
|---|---|---|
| Gini Index | $Gini = 1 - \frac{2}{N-1}\left(N - \frac{\sum_{i=1}^{N} i\tilde{y}_i}{\sum_{i=1}^{N} \tilde{y}_i}\right)$ where $\tilde{y}$ is the corresponding to $y$ after ranking the corresponding predicted values $\widehat{y}$. | Higher Gini is better. |
| Coefficient of Determination | $R^2 = 1 - \frac{\sum_{i=1}^{N}(\widehat{y}_i - y_i)^2}{\sum_{i=1}^{N}\left(y_i - \frac{1}{n}\sum_{i=1}^{n} y_i\right)^2}$ where $\widehat{y}$ is predicted values. | Higher $R^2$ is better. |
| Concordance Correlation Coefficient | $CCC = \frac{2\rho\sigma_{\widehat{y}_i}\sigma_{y_i}}{\sigma^2_{\widehat{y}_i} + \sigma^2_{y_i} + (\mu_{\widehat{y}_i} - \mu_{y_i})^2}$ where $\mu_{\widehat{y}_i}$ and $\mu_{y_i}$ are the means $\sigma^2_{\widehat{y}_i}$ and $\sigma^2_{y_i}$ are the variances $\rho$ is the correlation coefficient | Higher CCC is better. |
| Mean Error | $ME = \frac{1}{N}\sum_{i=1}^{N}(\widehat{y}_i - y_i)$ | Lower $\|ME\|$ is better. |
| Percentage Error | $PE = \frac{\sum_{i=1}^{N}\widehat{y}_i - \sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} y_i}$ | Lower $\|PE\|$ is better. |
| Mean Squared Error | $MSE = \frac{1}{N}\sum_{i=1}^{N}(\widehat{y}_i - y_i)^2$ | Lower $MSE$ is better |
| Mean Absolute Error | $MAE = \frac{1}{N}\sum_{i=1}^{N}\|\widehat{y}_i - y_i\|$ | Lower $MAE$ is better. |

# Appendix: Tuning hyperparameters

| R package | Description |
|---|---|
| `rpart` | Classification and regression tree (CART) |
| cp | complexity parameter |
| minsplit | minimum number of observations in a node in order to be considered for splitting |
| maxdepth | maximum depth of any node of the final tree |
| `randomForest` | Bagging and Random Forests |
| mtry | number of explanatory variables randomly sampled as candidates at each split |
| nodesize | minimum number of observations in the terminal nodes |
| ntree | number of trees to grow/bootstrap samples |
| `gbm` | Gradient boosting |
| n.trees | number of trees to fit/iterations/basis functions in the additive expansion |
| interaction.depth | maximum depth of variable interactions(1 implies an additive model, 2 means a model with up to 2-way interactions) |
| n.minobsinnode | minimum number of observations in the terminal nodes |
| shrinkage | shrinkage parameter(learning rate or step-size reduction) |
| `party/partykit` | Conditional inference trees |
| teststat | type of the test statistic to be applied for variable selection |
| splitstat | type of the test statistic to be applied for split point selection |
| testtype | the way to compute the distribution of the test statistic |
| alpha | significance level for variable selection |
| minsplit | minimum sum of weights in a node in order to be considered for splitting |
| `party/partykit` | Conditional random forests |
| mtry | number of explanatory variables randomly sampled as candidates at each split |
| ntree | number of trees to grow/bootstrap samples |

# References


- Breiman, L., et al. (1984). *Classification and Regression Trees*. Taylor & Francis Group, LLC: Boca Raton, FL.
- Gan, G. and Valdez, E.A. (2019). *Metamodeling for Variable Annuities*. CRC Press: Boca Raton, FL.
- Gan, G. and Valdez, E.A. (2017). Valuation of large variable annuity portfolios: Monte Carlo simulation and synthetic datasets. *Dependence Modeling*. 5:354-374.
- Gan, G. and Valdez, E.A. (2018). Regression modeling for the valuation of large variable annuity portfolios. *North American Actuarial Journal*. 22(1):40-54.
- Hothorn, T., Hornik, K. and Zeileis, A. (2006). Unbiased recursive partitioning: A conditional inference framework. *Journal of Computational and Graphical Statistics*. 15(3):651-674.
- Loh, W.-Y. (2014). Fifty years of classification and regression trees. *International Statistical Review*. 82(3):329-348.
- Quan, Z., Gan, G. and Valdez, E.A. (2019). Tree-based models for variable annuity valuation: Parameter tuning and empirical analysis. Submitted for publication.