Insurance Pricing in a Competitive Market

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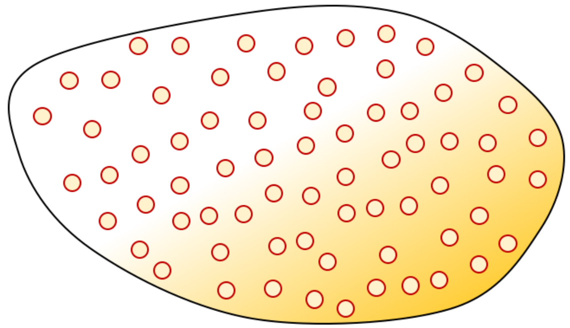
Insurance, "segmentation" & "mutualization"

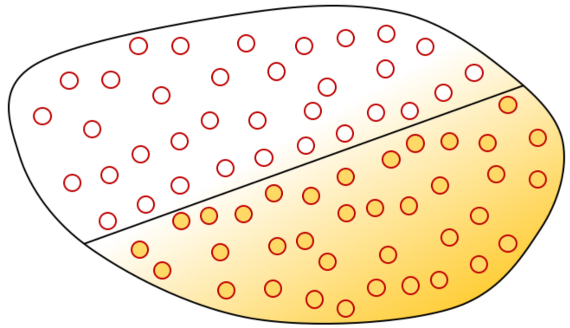
"*Insurance is the contribution of the many to the misfortune of the few* "

What should be that "*contribution*" ? Premium



mutualization (of risk) : *spread of risk among several parties* see pooling, sharing

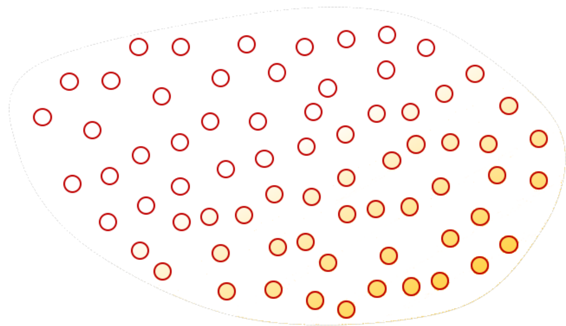
$\pi = \mathbb{E}_{\mathbb{P}}[S_1]$



(market) segmentation : *division of a market into identifiable groups* see differentiation, customization

$\pi(\omega) = \mathbb{E}_{\mathbb{P}}[S_1 | \Omega = \omega]$

for some (unobservable) risk factor Ω



Use of features (covariates) $\boldsymbol{x}$ as a proxy

$\pi(\boldsymbol{x}) = \mathbb{E}_{\mathbb{P}}[S_1 | \boldsymbol{X} = \boldsymbol{x}] = \mathbb{E}_{\mathbb{P}_{\boldsymbol{x}}}[S_1]$

Risk Transfert without Segmentation

	Insured	Insurer
Loss	$\mathbb{E}[S]$	$S - \mathbb{E}[S]$
Average Loss	$\mathbb{E}[S]$	0
Variance	0	$\text{Var}[S]$

All the risk - $\text{Var}[S]$ - is kept by the insurance company.

Remark: all those interpretation are discussed in Denuit & Charpentier (2004).

Risk Transfert with Segmentation and Perfect Information

Assume that information $\boldsymbol{\Omega}$ is observable,

	Insured	Insurer
Loss	$\mathbb{E}[S\vert\boldsymbol{\Omega}]$	$S-\mathbb{E}[S\vert\boldsymbol{\Omega}]$
Average Loss	$\mathbb{E}[S]$	0
Variance	$\text{Var}\Big[\mathbb{E}[S\vert\boldsymbol{\Omega}]\Big]$	$\text{Var}\Big[S-\mathbb{E}[S\vert\boldsymbol{\Omega}]\Big]$

Observe that $\text{Var}\Big[S-\mathbb{E}[S|\boldsymbol{\Omega}]\Big]=\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]$, so that

$$\text{Var}[S]=\underbrace{\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]}_{\rightarrow\text{ insurer}}+\underbrace{\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big]}_{\rightarrow\text{ insured}}.$$

Risk Transfert with Segmentation and Imperfect Information

Assume that $\boldsymbol{X} \subset \boldsymbol{\Omega}$ is observable

	Insured	Insurer
Loss	$\mathbb{E}[S\|\boldsymbol{X}]$	$S - \mathbb{E}[S\|\boldsymbol{X}]$
Average Loss	$\mathbb{E}[S]$	0
Variance	$\text{Var}\big[\mathbb{E}[S\|\boldsymbol{X}]\big]$	$\mathbb{E}\big[\text{Var}[S\|\boldsymbol{X}]\big]$

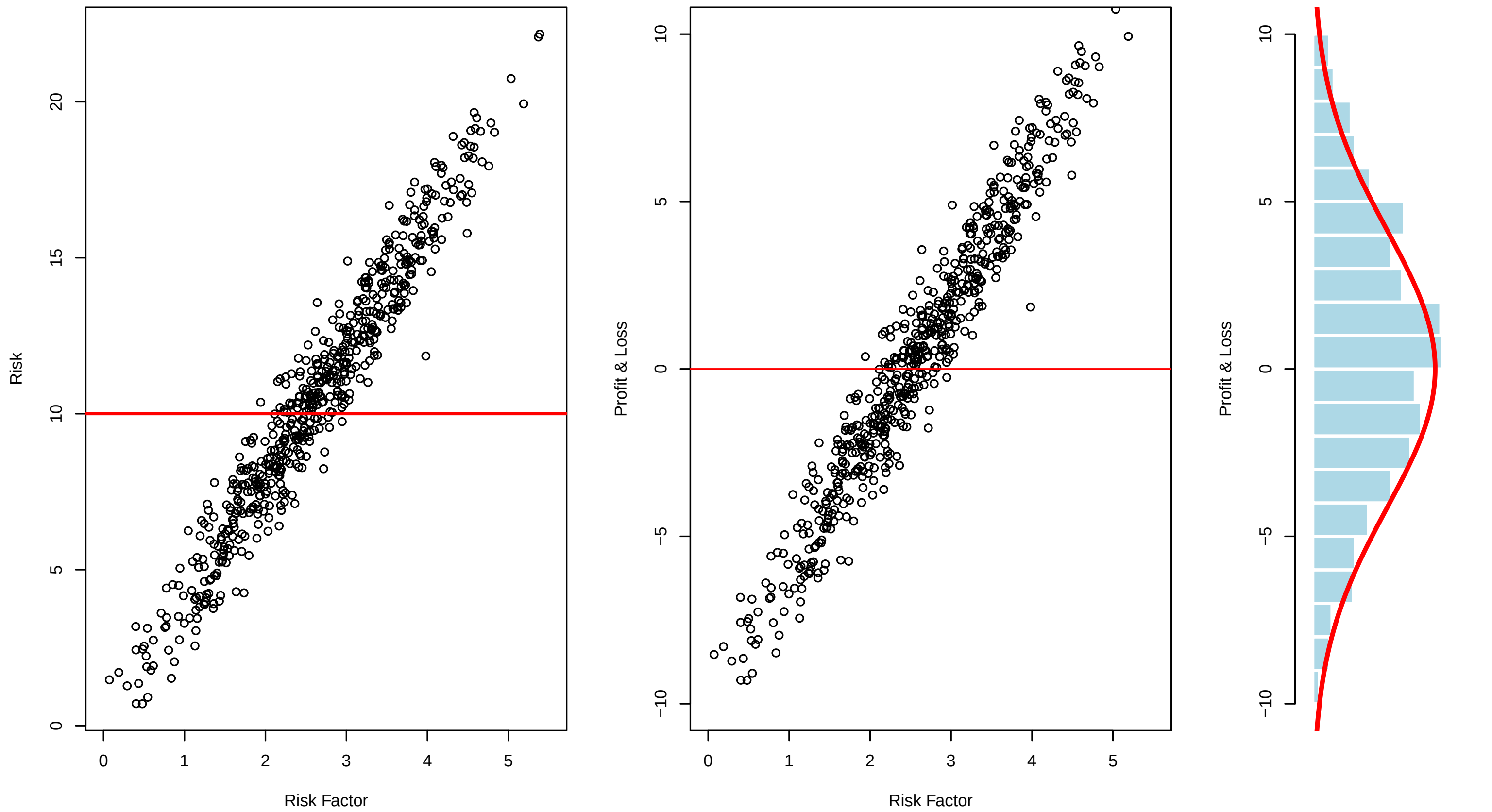
Now

$$
\begin{aligned}
\mathbb{E}\Big[\text{Var}[S|\boldsymbol{X}]\Big] &= \mathbb{E}\Big[\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big] + \mathbb{E}\Big[\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big] \\
&= \underbrace{\mathbb{E}\Big[\text{Var}[S|\boldsymbol{\Omega}]\Big]}_{\text{perfect pricing}} + \underbrace{\mathbb{E}\Big\{\text{Var}\Big[\mathbb{E}[S|\boldsymbol{\Omega}]\Big|\boldsymbol{X}\Big]\Big\}}_{\text{misfit}}.
\end{aligned}
$$

spiral of segmentation...

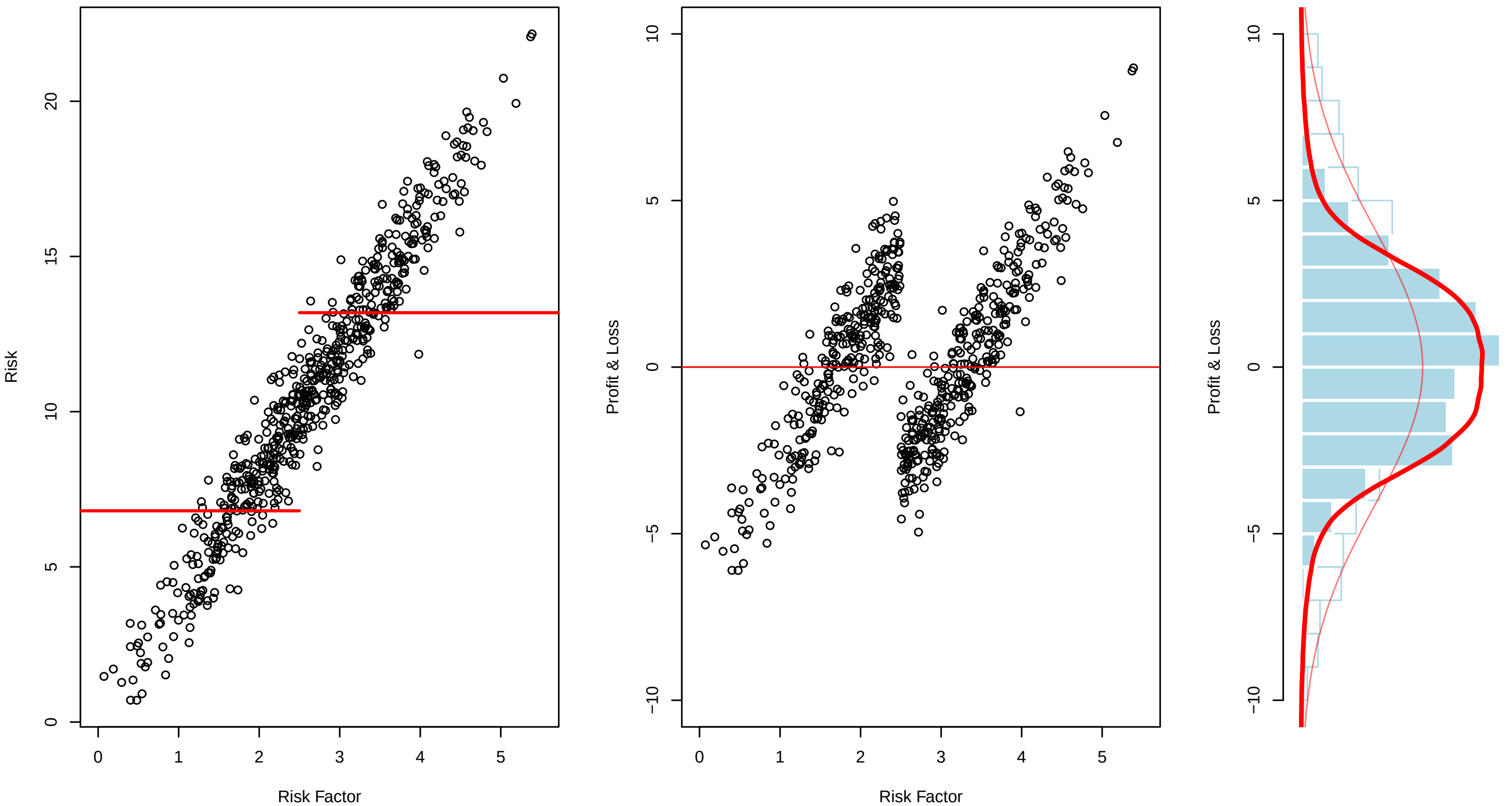
Risk Transfert : Visualization

Pricing without Segmentation : $\pi(x) = \mathbb{E}[Y]$ (whatever x)



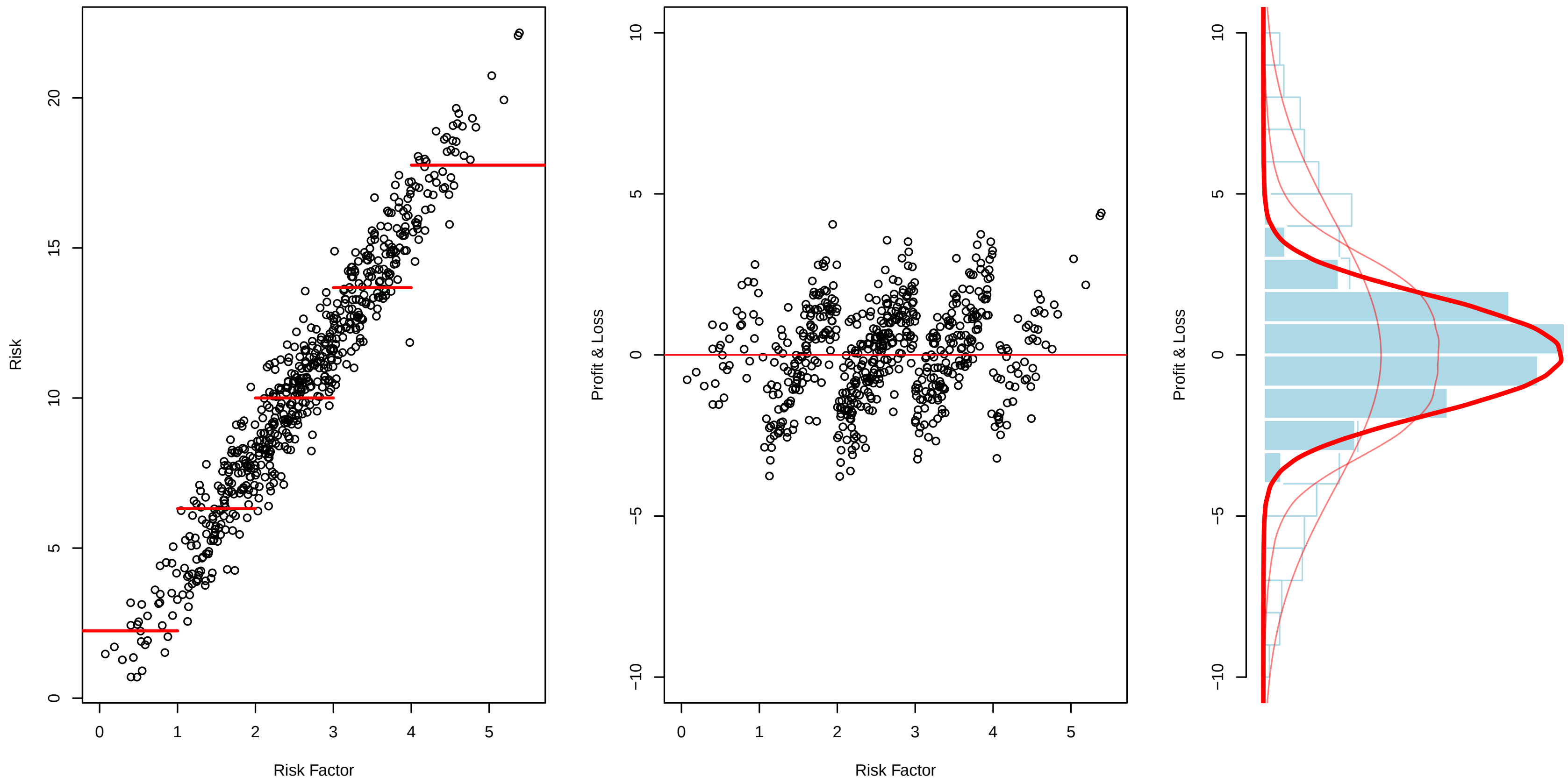
Risk Transfert : Visualization

Pricing with Imperfect Segmentation : $\pi(x) = \mathbb{E}[Y|x] = \begin{cases} \overline{\pi} \text{ if } x > s \\ \underline{\pi} \text{ if } x \leq s \end{cases}$



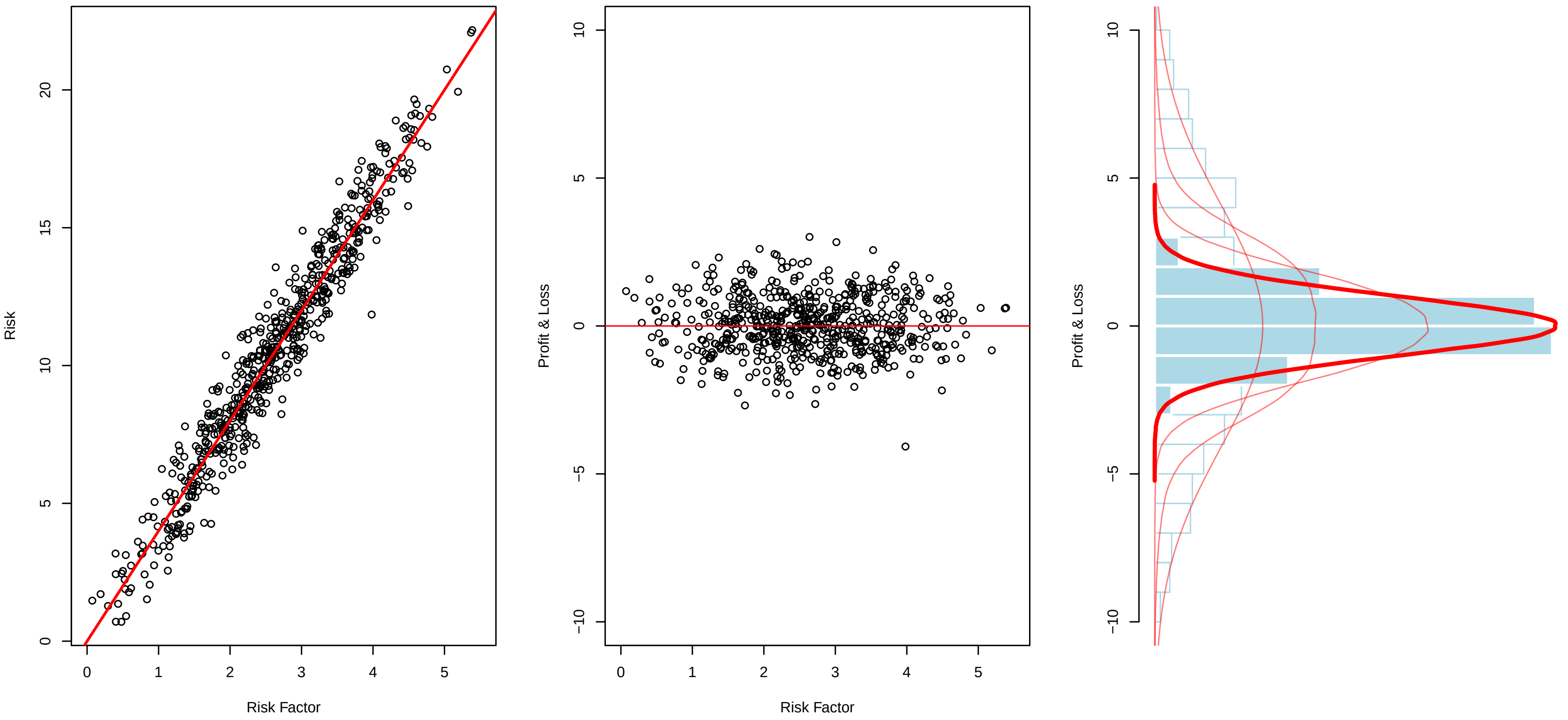
Risk Transfert : Visualization

Pricing with Imperfect Segmentation : $\pi(x) = \mathbb{E}[Y|x] = \pi_{\lfloor x \rfloor}$



Risk Transfert : Visualization

Pricing with Perfect Segmentation : $\pi(x) = \mathbb{E}[Y|x]$ with $x = \omega$.



Actuarial Pricing Model

Premium is $\mathbb{E}\big[S|\boldsymbol{X}=\boldsymbol{x}\big]=\mathbb{E}\left[\sum_{i=1}^{N}Y_i\middle|\boldsymbol{X}=\boldsymbol{x}\right]=\mathbb{E}\left[N|\boldsymbol{X}=\boldsymbol{x}\right]\cdot\mathbb{E}\left[Y|\boldsymbol{X}=\boldsymbol{x}\right]$

Statistical and modeling issues to approximate based on some training datasets, with claims frequency $\{n_i,\boldsymbol{x}_i\}$ and individual losses $\{y_i,\boldsymbol{x}_i\}$.

Use GLM to approximate $\mathbb{E}\left[N|\boldsymbol{X}=\boldsymbol{x}\right]$ and $\mathbb{E}\left[Y|\boldsymbol{X}=\boldsymbol{x}\right]$

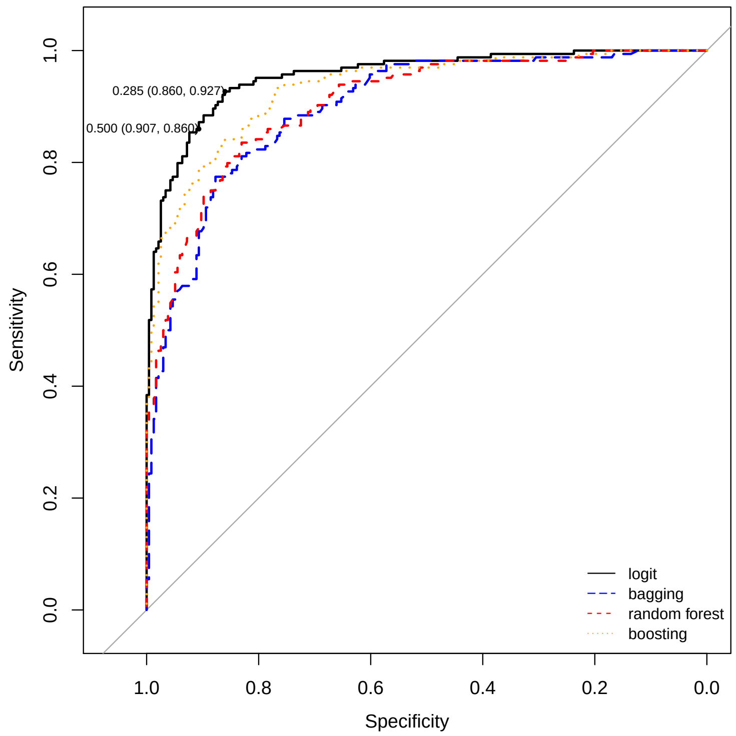
Recall that $\mathbb{E}\Big[\mathbb{E}\big[S|\boldsymbol{X}\big]\Big]=\mathbb{E}\big[S\big]$

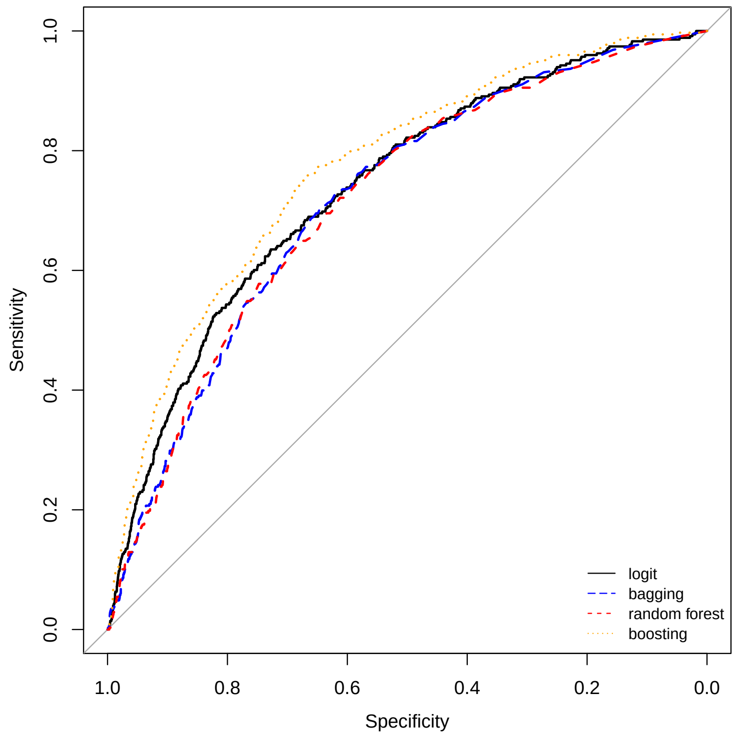
How can we claim that model $\widehat{\pi}(\boldsymbol{x})=\widehat{\mathbb{E}}\big[S|\boldsymbol{X}=\boldsymbol{x}\big]$ is “good” ?

How can we visualize the goodness of a model ?

In the context of classification, use ROC curve

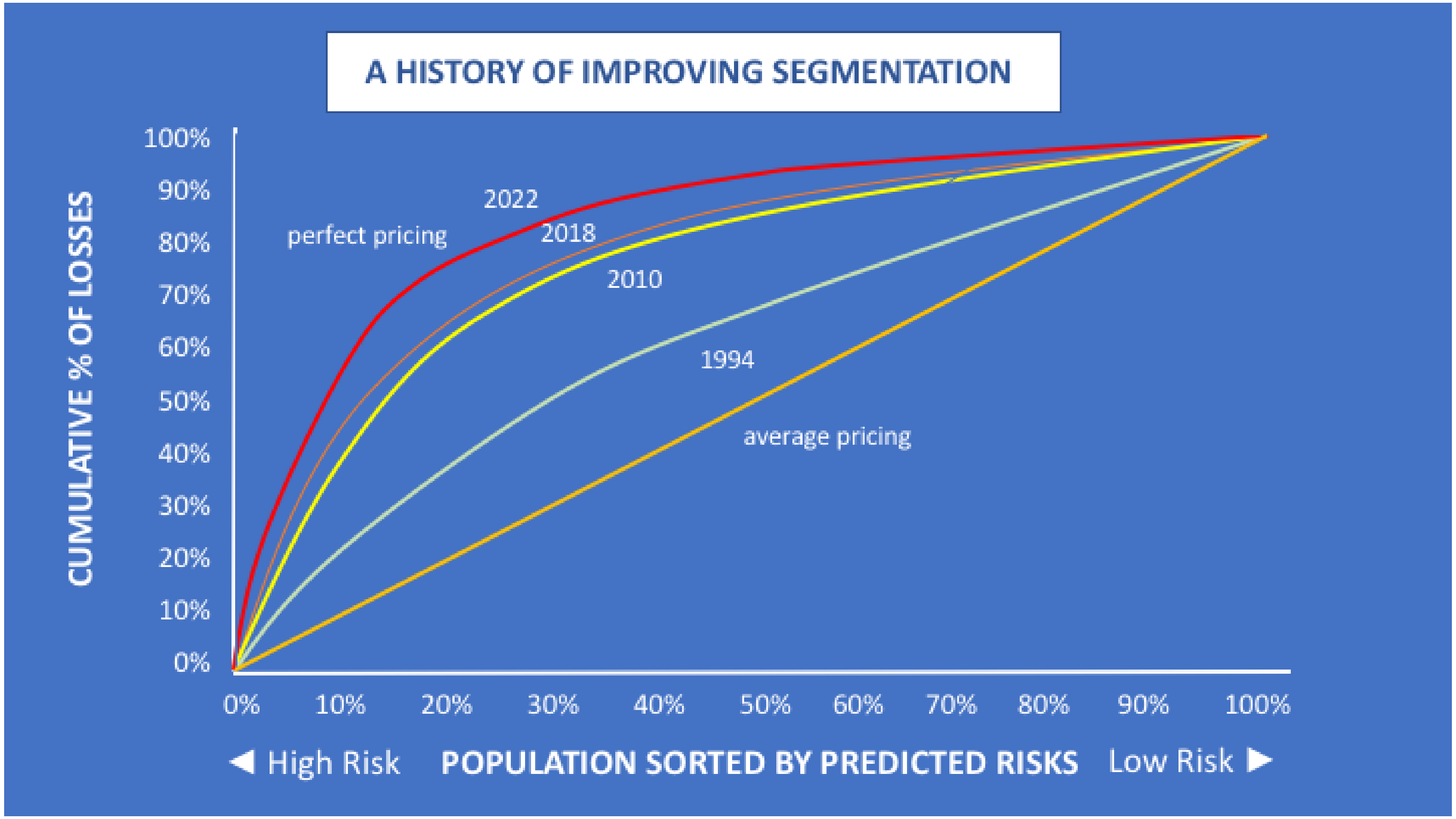
$$\text{sensitivity} = \text{TNR} = \frac{\text{TN}}{\text{TN}+\text{FP}} = \mathbb{P}[\underbrace{m(\boldsymbol{X}) > s}_{\widehat{Y}=1}|Y=1] \text{ vs. } \text{specificity} = \text{TPR} = \frac{\text{TP}}{\text{TP}+\text{FN}}$$





Source : Charpentier, Flachaire & Ly (2018)

How can we visualize the goodness of a model ?

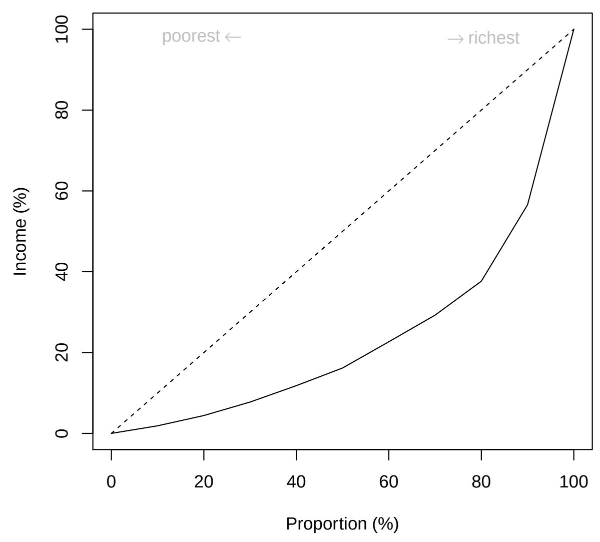


Source : https://www.progressive.com/jobs/analyst-program/

Model Comparison and Lorenz curves

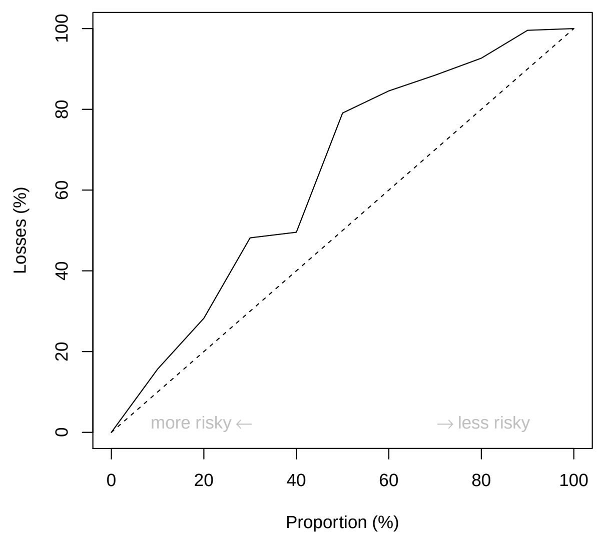
Consider an ordered sample $\{y_1, \cdots, y_n\}$ of incomes, with $y_1 \leq y_2 \leq \cdots \leq y_n$, then Lorenz curve is

$$\{F_i, L_i\} \text{ with } F_i = \frac{i}{n} \text{ and } L_i = \frac{\sum_{j=1}^{i} y_j}{\sum_{j=1}^{n} y_j}$$



We have observed losses y_i and premiums $\widehat{\pi}(\boldsymbol{x}_i)$. Consider an ordered sample by the model, see Frees, Meyers & Cummins (2014), $\widehat{\pi}(\boldsymbol{x}_1) \geq \widehat{\pi}(\boldsymbol{x}_2) \geq \cdots \geq \widehat{\pi}(\boldsymbol{x}_n)$, then plot

$$\{F_i, L_i\} \text{ with } F_i = \frac{i}{n} \text{ and } L_i = \frac{\sum_{j=1}^{i} y_j}{\sum_{j=1}^{n} y_j}$$



Model Comparison for Life Insurance Models

Consider the case of a death insurance contract, that pays 1 if the insured deceased within the year.

$\pi(x) = \mathbb{E}\big[T_x \leq t+1 | T_x > t\big]$

— No price discrimination $\pi = \mathbb{E}[\pi(X)]$

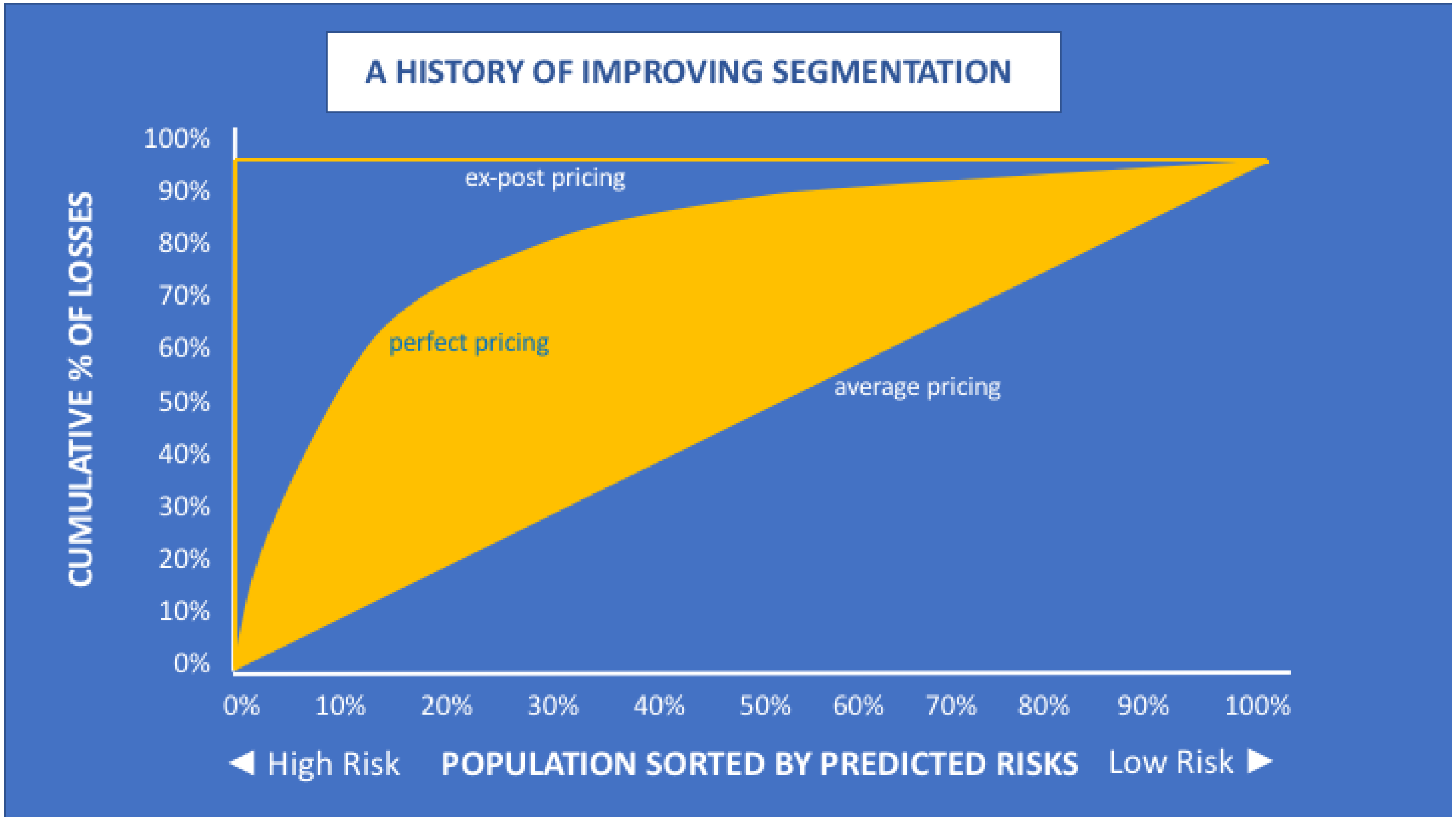
— Perfect discrimination $\pi(x)$

— Imperfect discrimination

$\pi_- = \mathbb{E}[\pi(X)|X < s]$ and $\pi_+ = \mathbb{E}[\pi(X)|X > s]$

click to visualize the construction

How to understand this (*pseudo*)-Lorenz curve ?

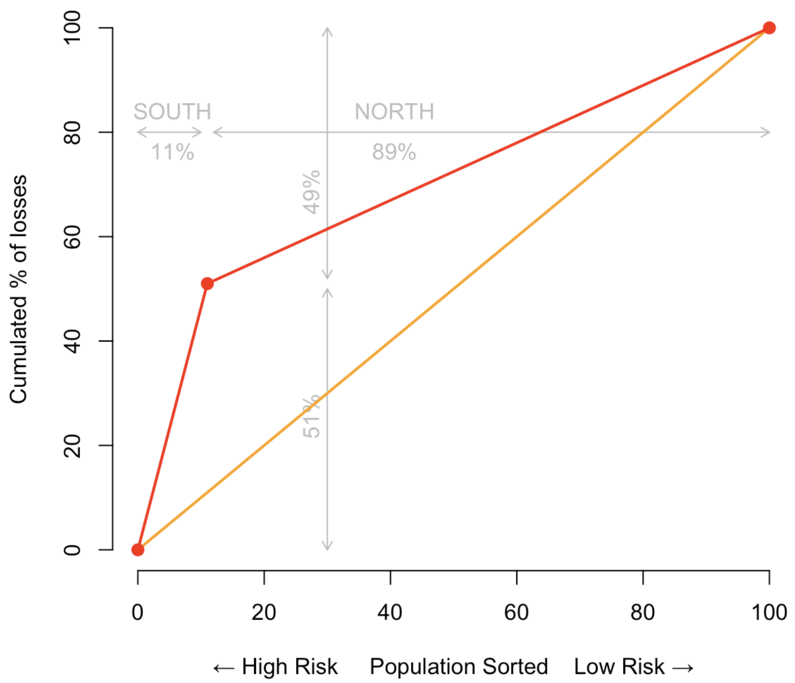


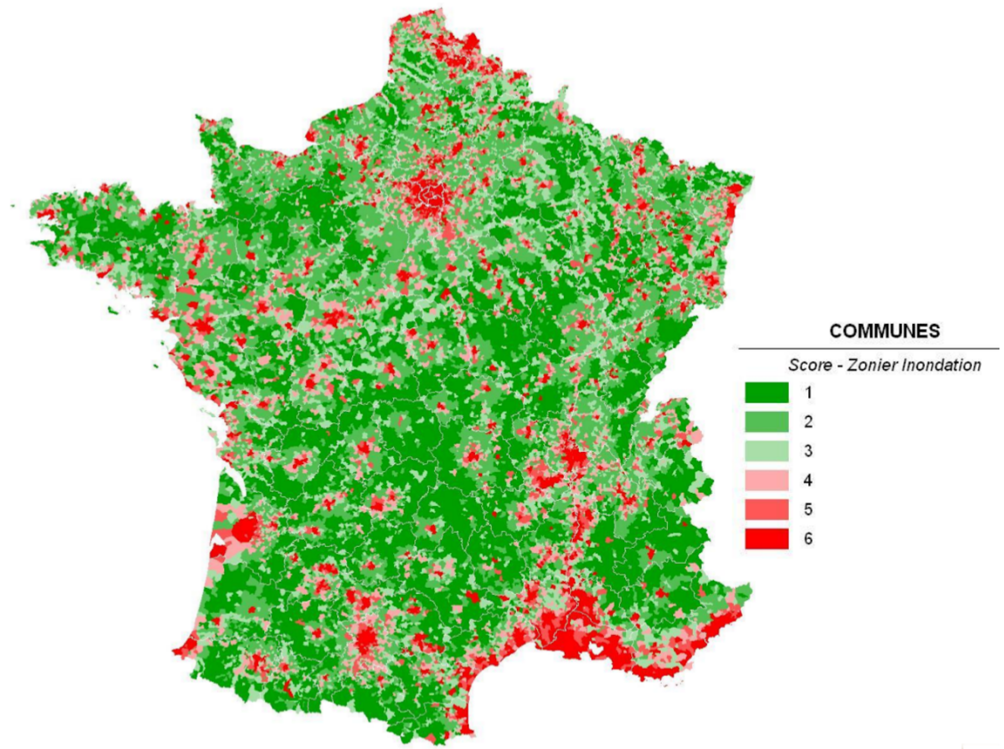
Where is the upper bound ?

Insurance, Risk Pooling and Solidarity

Consider flood risk, in France

One can look at the “Lorenz curve”





Price Differentiation, a Toy Example

Claims frequency $N \in \{0,1\}$ (average cost = 1,000)

		X_1			
		Young	Experienced	Senior	Total
X_2	Town	12% (500)	9% (2,000)	9% (500)	9.5% (3,000)
	Outside	8% (500)	6.67% (1,000)	4% (500)	6.33% (2,000)
	Total	10% (1,000)	8.22% (3,000)	6.5% (1,000)	8.23% (5,000)

from Charpentier, Denuit & Élie (2015)

Price Differentiation, a Toy Example

	Y-T (500)	Y-O (500)	E-T (2,000)	E-O (1,000)	S-T (500)	S-O (500)
none	82.3	82.3	82.3	82.3	82.3	82.3
$X_1 \times X_2$	120	80	90	66.7	90	40
market	82.3	80	82.3	66.7	82.3	40

none	82.3	82.3	82.3	82.3	82.3	82.3
X_1	100	100	82.2	82.2	65	65
X_2	95	63.3	95	63.3	95	63.3
$X_1 \times X_2$	120	80	90	66.7	90	40
market	82.3	63.3	82.2	63.3	65	40

Price Differentiation, a Toy Example

	premium	losses	loss ratio		99.5% quantile	Market Share
none	247	285	115.4%	(±8.9%)	132%	66.1%
$X_1 \times X_2$	126.67	126.67	100.0%	(±10.4%)	123%	33.9%
market	373.67	411.67	110.2%	(±5.1%)	123%	100.0%

none	41.17	60	145.7%	(±34.6%)	189%	11.6%
X_1	196.94	225	114.2%	(±11.8%)	140%	55.8%
X_2	95	106.67	112.3%	(±15.1%)	134%	26.9%
$X_1 \times X_2$	20	20	100.0%	(±41.9%)	160%	5.7%
market	353.10	411.67	116.6%	(±5.3%)	130%	100.0%

From Econometric to 'Machine Learning' Techniques

In a competitive market, insurers can use different sets of variables and different models, e.g. GLMs, $N_t|\boldsymbol{X} \sim \mathcal{P}(\lambda_{\boldsymbol{X}} \cdot t)$ and $Y|\boldsymbol{X} \sim \mathcal{G}(\mu_{\boldsymbol{X}}, \varphi)$

$$\widehat{\pi}_j(\boldsymbol{x}) = \widehat{\mathbb{E}}\big[N_1\big|\boldsymbol{X} = \boldsymbol{x}\big] \cdot \widehat{\mathbb{E}}\big[Y\big|\boldsymbol{X} = \boldsymbol{x}\big] = \underbrace{\exp(\widehat{\boldsymbol{\alpha}}^\mathsf{T}\boldsymbol{x})}_{\text{Poisson } \mathcal{P}(\lambda_{\boldsymbol{x}})} \cdot \underbrace{\exp(\widehat{\boldsymbol{\beta}}^\mathsf{T}\boldsymbol{x})}_{\text{Gamma } \mathcal{G}(\mu_{\boldsymbol{X}},\varphi)}$$

that can be extended to GAMs,

$$\widehat{\pi}_j(\boldsymbol{x}) = \underbrace{\exp\left(\sum_{k=1}^{d} \widehat{s}_k(x_k)\right)}_{\text{Poisson } \mathcal{P}(\lambda_{\boldsymbol{x}})} \cdot \underbrace{\exp\left(\sum_{k=1}^{d} \widehat{t}_k(x_k)\right)}_{\text{Gamma } \mathcal{G}(\mu_{\boldsymbol{X}},\varphi)}$$

or some Tweedie model on S_t (compound Poisson, see Tweedie (1984)) conditional on $\boldsymbol{X}$

From Econometric to 'Machine Learning' Techniques

(see Charpentier & Denuit (2005) or Kaas *et al.* (2008)) or any other statistical model

$$\widehat{\pi}_j(\boldsymbol{x}) \text{ where } \widehat{\pi}_j \in \underset{m\in\mathcal{F}_j:\mathcal{X}_j\rightarrow\mathbb{R}}{\text{argmin}}\left\{\sum_{i=1}^{n}\ell(s_i,m(\boldsymbol{x}_i))\right\}$$

For some loss function $\ell:\mathbb{R}^2\rightarrow\mathbb{R}_+$ (usually an L_2 based loss, $\ell(s,y)=(s-y)^2$ since $\text{argmin}\{\mathbb{E}[\ell(S,m)],\ m\in\mathbb{R}\}$ is $\mathbb{E}(S)$, interpreted as the pure premium).

For instance, consider regression trees, forests, neural networks, or boosting based techniques to approximate $\pi(\boldsymbol{x})$, and various techniques for variable selection, such as LASSO (see Hastie *et al.* (2009) or Charpentier *et al.* (2017) for a description and a discussion).

With d competitors, each insured i has to choose among d premiums, $\boldsymbol{\pi}_i=\big(\widehat{\pi}_1(\boldsymbol{x}_i),\cdots,\widehat{\pi}_d(\boldsymbol{x}_i)\big)\in\mathbb{R}_+^d$.

Machine Learning & Credit

Before discussing the use of those models in insurance, note that the same issues exist in credit, see Hardt, Price & Srebro (2017).

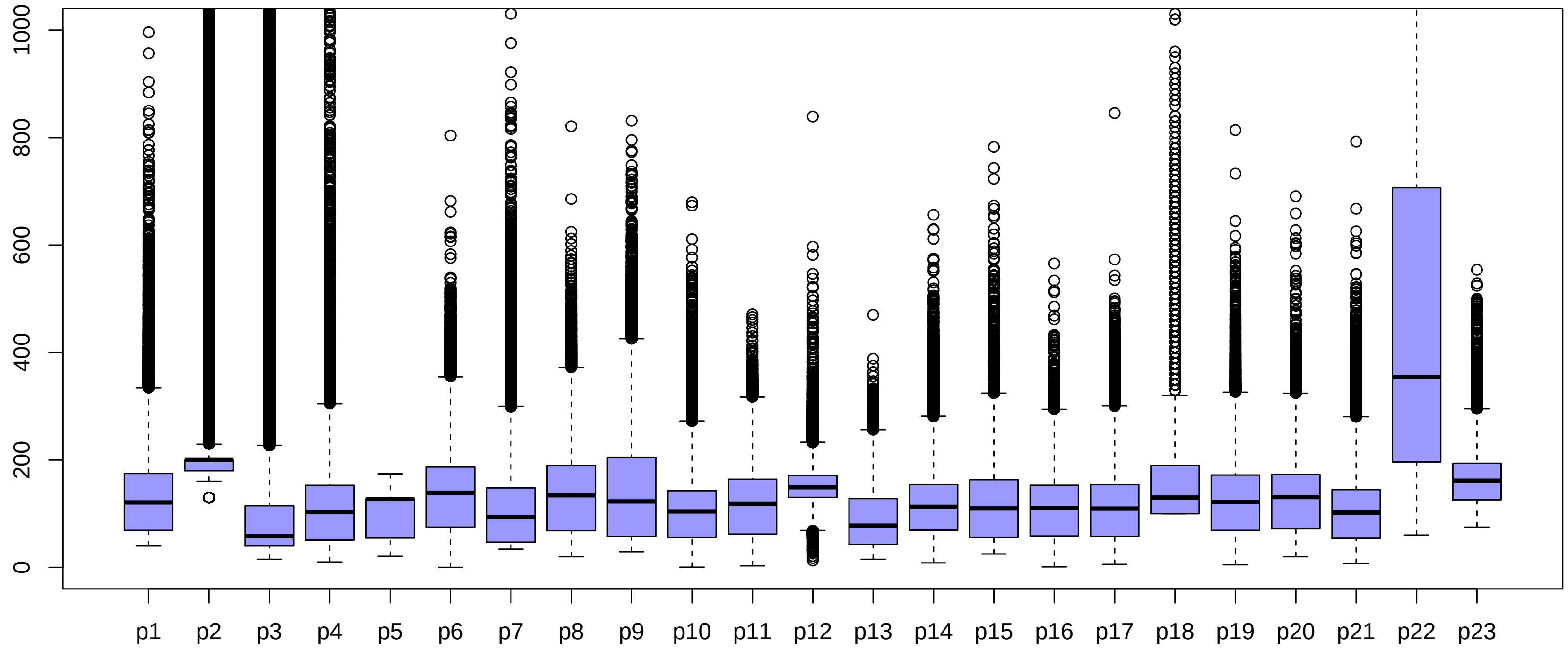
"*the shift from traditional to machine learning lending models may have important distributional effects for consumers* [...] *machine learning would offer lower rates to racial groups who already were at an advantage under the traditional model, but it would also benefit disadvantaged groups by enabling them to obtain a mortgage in the first place* " Fuster, Goldsmith-Pinkham, Ramadorai & Walther (2017)

Field experiment: the actuarial pricing games

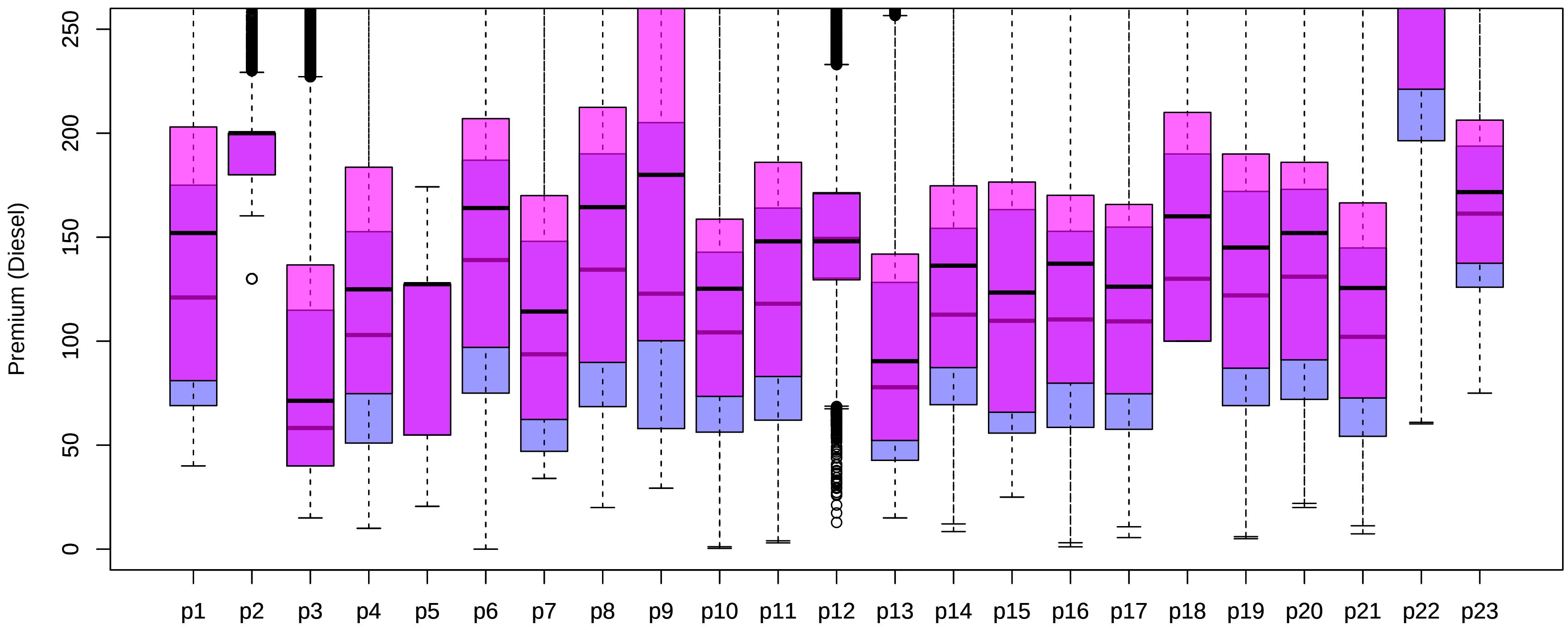
Actuarial pricing is data based, and model based

To understand how model influence pricing
we ran some actuarial pricing games

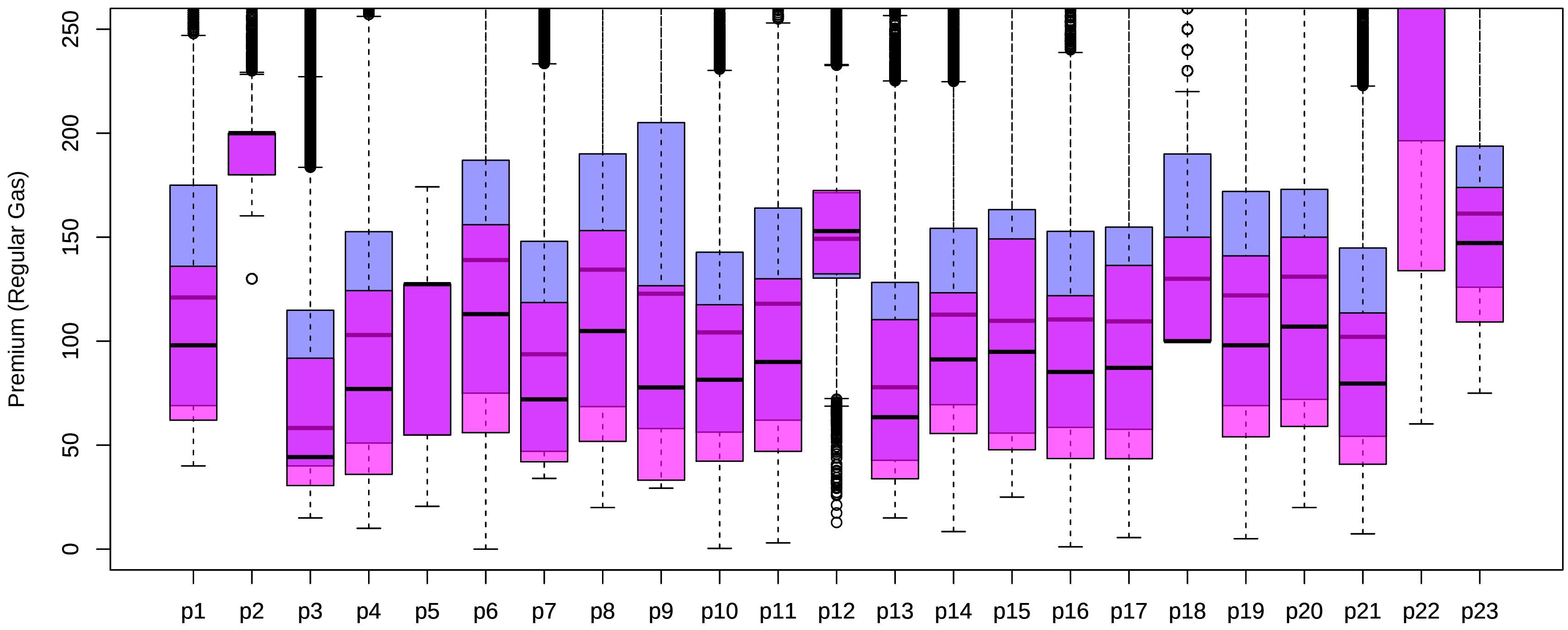
Insurance Ratemaking Before Competition



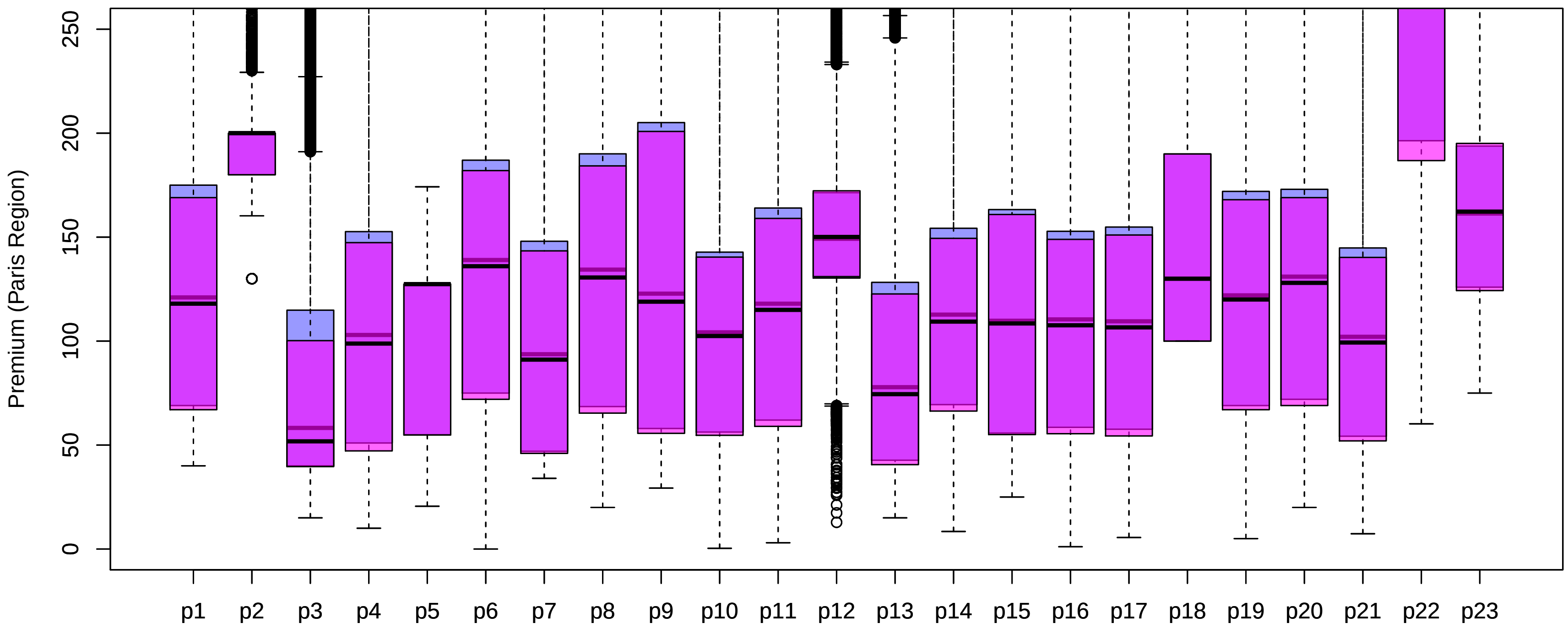
Insurance Ratemaking Before Competition Gas Type Diesel



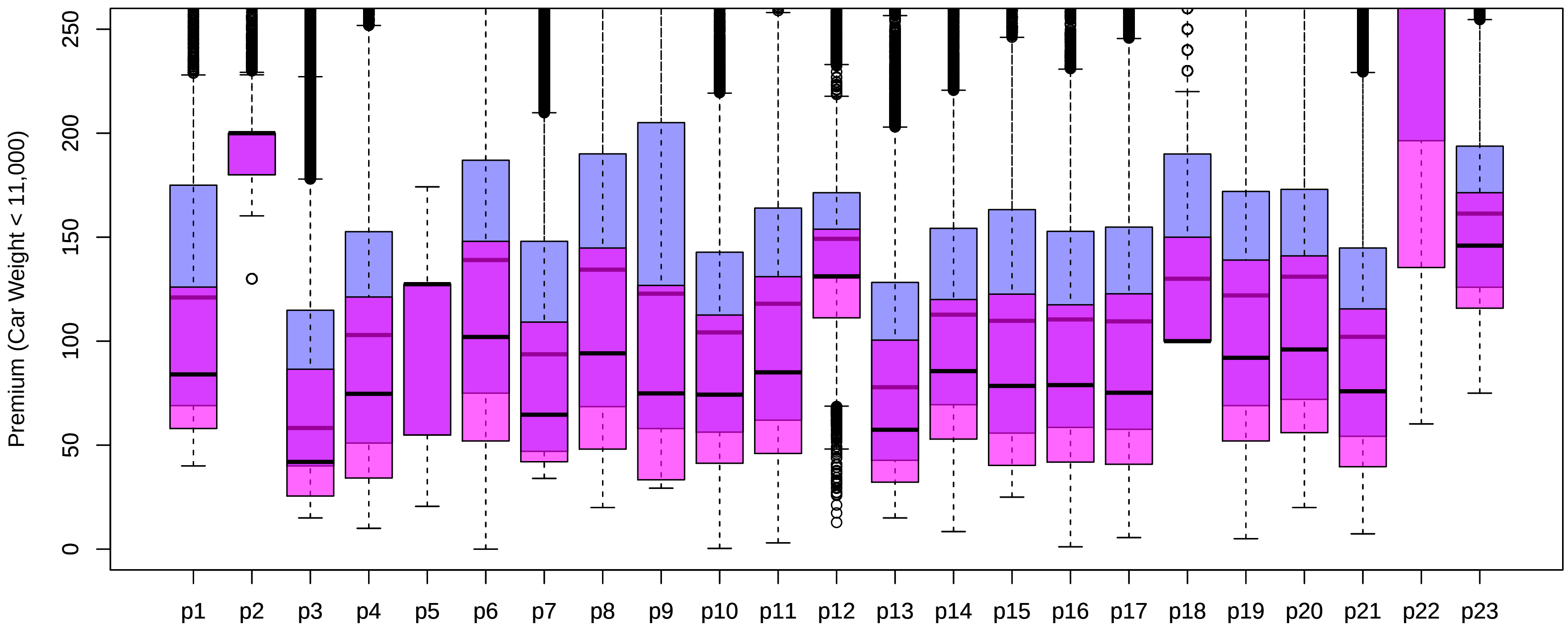
Insurance Ratemaking Before Competition Gas Type Regular



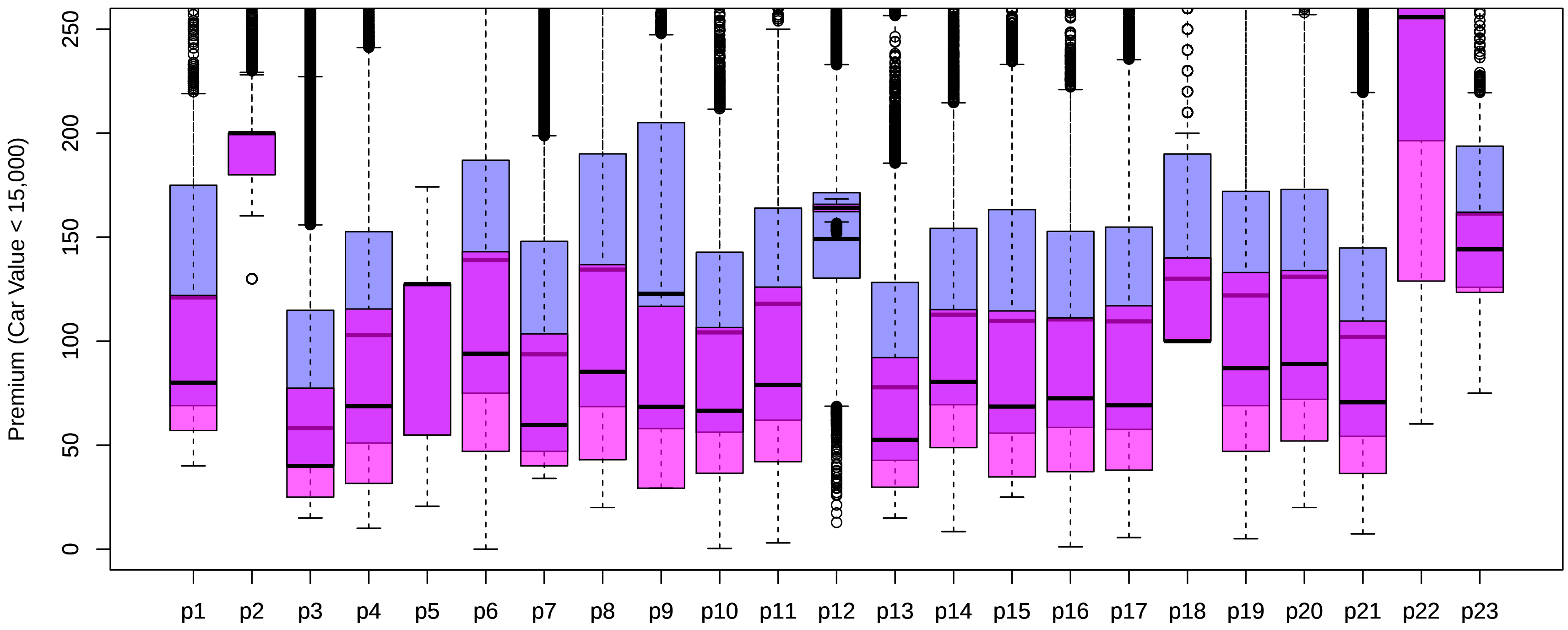
Insurance Ratemaking Before Competition Paris Region



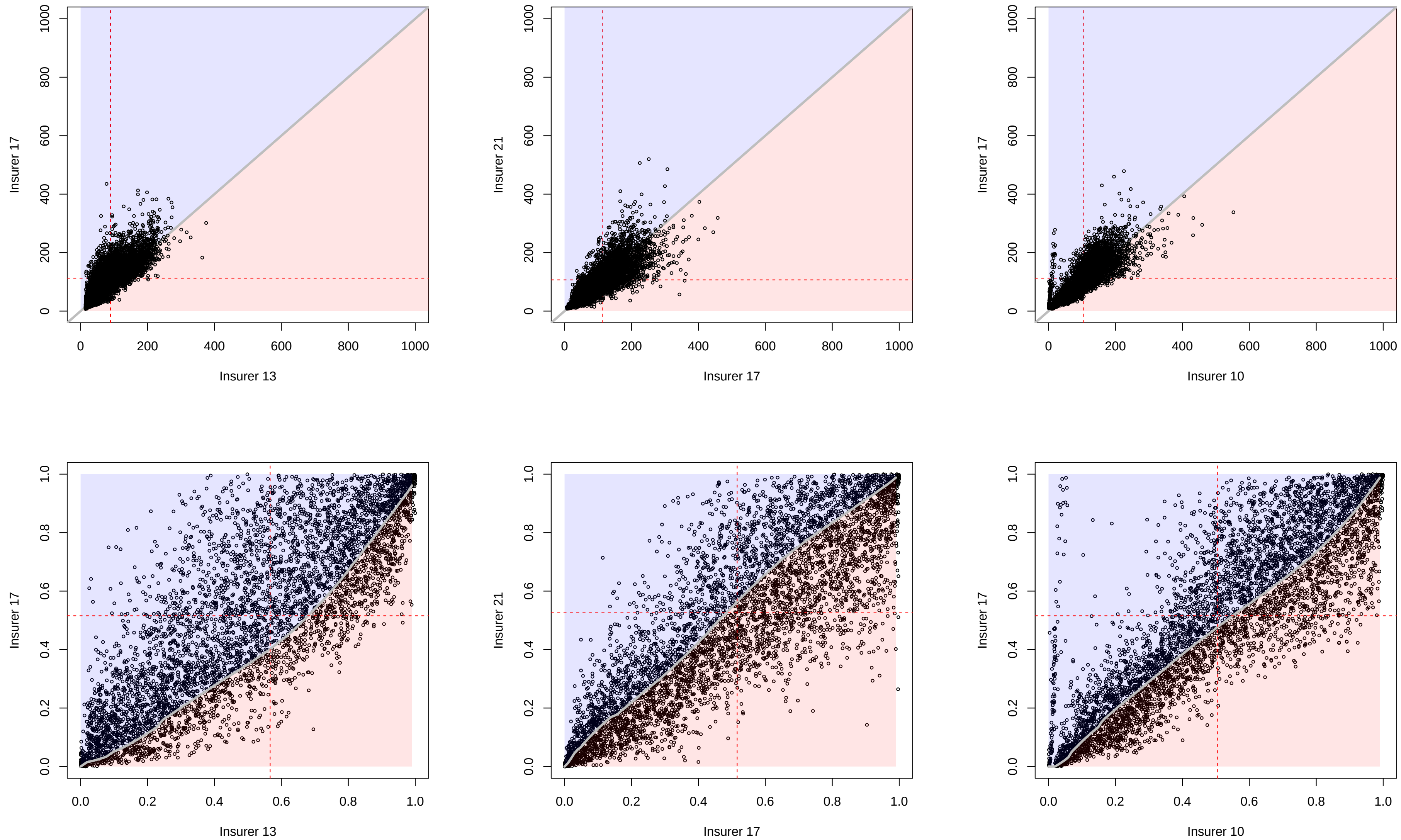
Insurance Ratemaking Before Competition Car Weight



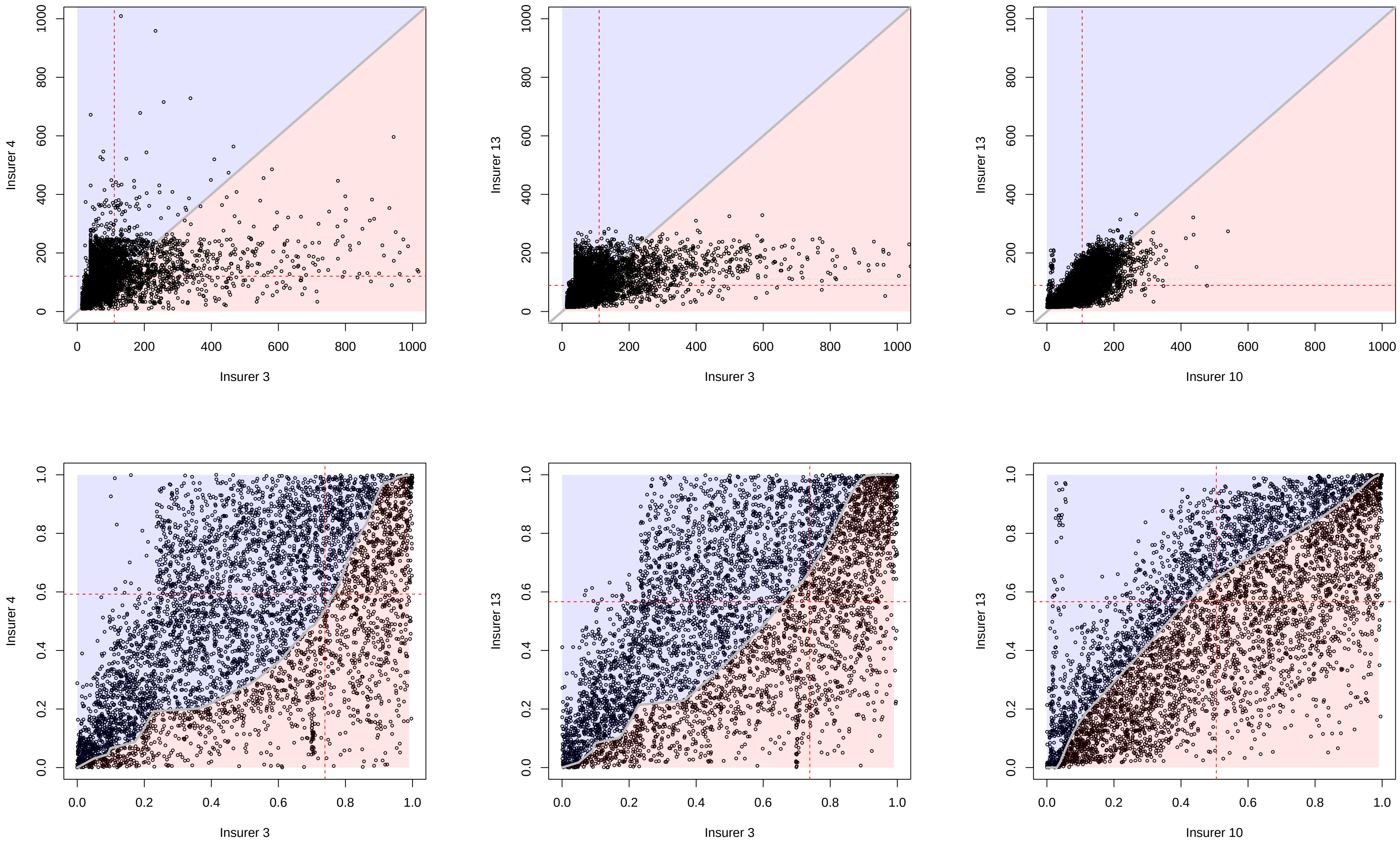
Insurance Ratemaking Before Competition Car Value



Insurance Ratemaking Competition : High Correlation ?



Insurance Ratemaking Competition : High Correlation ?



Insurance Ratemaking Competition

We need a **Decision Rule** to select premium chosen by insured i

	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
	473.15	447.58	343.64	410.76	414.23	425.23
	337.98	336.20	468.45	339.33	383.55	672.91

Insurance Ratemaking Competition

Basic '**rational rule**' $\pi_i = \min\{\widehat{\pi}_1(\boldsymbol{x}_i), \cdots, \widehat{\pi}_d(\boldsymbol{x}_i)\} = \widehat{\pi}_{1:d}(\boldsymbol{x}_i)$

	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
	473.15	447.58	343.64	410.76	414.23	425.23
	337.98	336.20	468.45	339.33	383.55	672.91

Insurance Ratemaking Competition

A more **realistic rule** $\pi_i \in \{\widehat{\pi}_{1:d}(\boldsymbol{x}_i), \widehat{\pi}_{2:d}(\boldsymbol{x}_i), \widehat{\pi}_{3:d}(\boldsymbol{x}_i)\}$

	Ins1	Ins2	Ins3	Ins4	Ins5	Ins6
	787.93	706.97	1032.62	907.64	822.58	603.83
	170.04	197.81	285.99	212.71	177.87	265.13
	473.15	447.58	343.64	410.76	414.23	425.23
	337.98	336.20	468.45	339.33	383.55	672.91

A Game with Rules... but no (explicit) Goal

Two datasets : a training one, and a pricing one (without the losses in the later)

Step 1 : provide premiums to all contracts in the same pricing dataset

rescaling $\widetilde{\pi}_j(\boldsymbol{x}) = \alpha_j \widehat{\pi}_j(\boldsymbol{x})$

such that $\displaystyle\sum_{i \in \text{pricing}} \widetilde{\pi}_j(\boldsymbol{x}_i)$ is identical, $\forall j$

Step 2 : allocate insured among players

Season 1 13 players

Season 2 14 players

Step 3 [season 2] : provide additional information (premiums of competitors)

Season 3 23 players (3 markets, 8+8+7)

Step 3-6 [season 3] : dynamics, 4 years

A Game with Rules... Forthcoming versions

Two datasets : a training one $(\boldsymbol{x}_i, y_i)$, and a pricing one $(\boldsymbol{x}_i)$

Step 1 : provide a pricing model (.r or .py)

Step 1a : validate a small subset of the pricing one

Step 1b : validate $\displaystyle\sum_{i\in\text{training}_j} \widetilde{\pi}_j(\boldsymbol{x}_i) \geq \sum_{i\in\text{training}_j} y_i$

validate $\displaystyle\sum_{i\in\text{training}_j, x_i\in\mathcal{S}} \widetilde{\pi}_j(\boldsymbol{x}_i) \geq \sum_{i\in\text{training}_j, x_i\in\mathcal{S}} y_i$ for some set $\mathcal{S}$

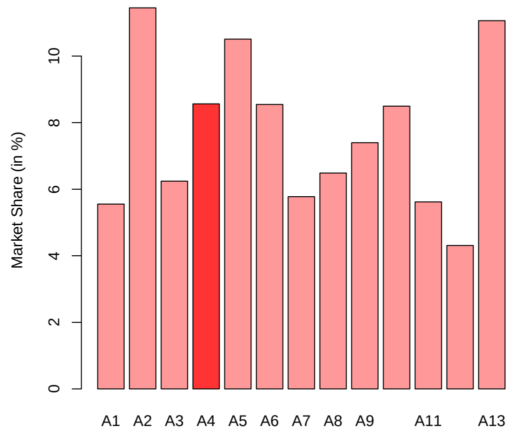
Step 2 : allocate insured among players

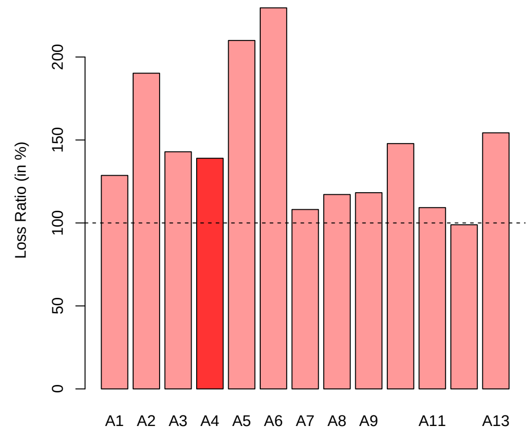
Pricing Game in 2015

Insurer 4

GLM for frequency and standard cost (large claimes were removed, above 15k), Interaction Age and Gender

Actuary working for a *mutuelle* company

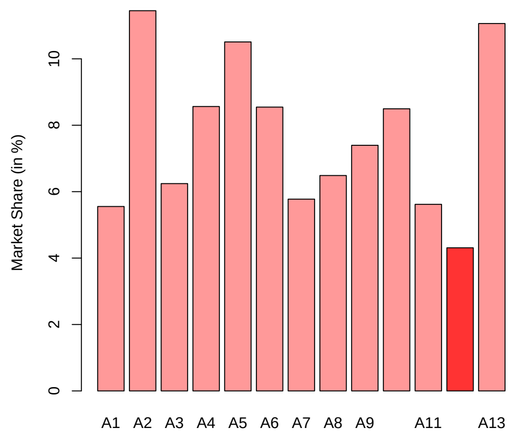


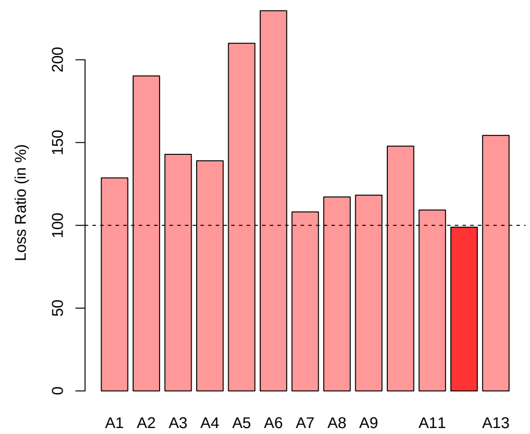


Insurer 11

Use of two XGBoost models (bodily injury and material), with correction for negative premiums

Actuary working for a private insurance company



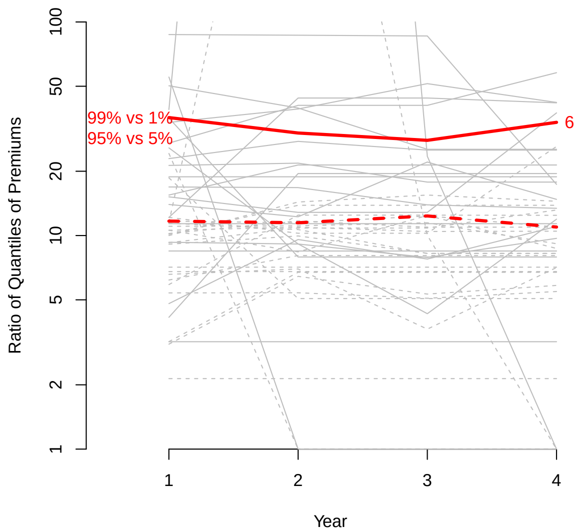


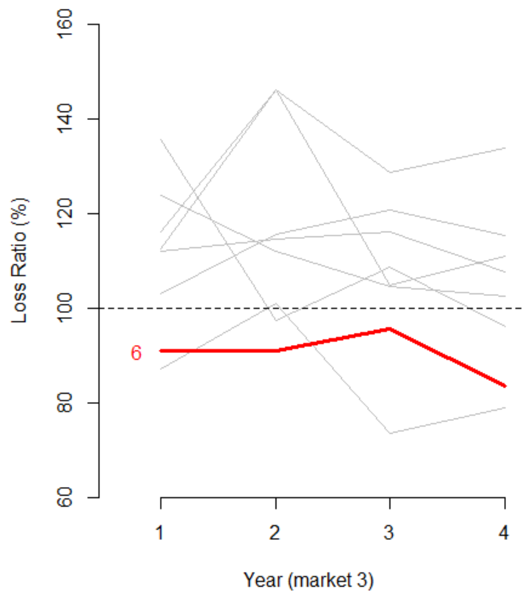
Pricing Game in 2017

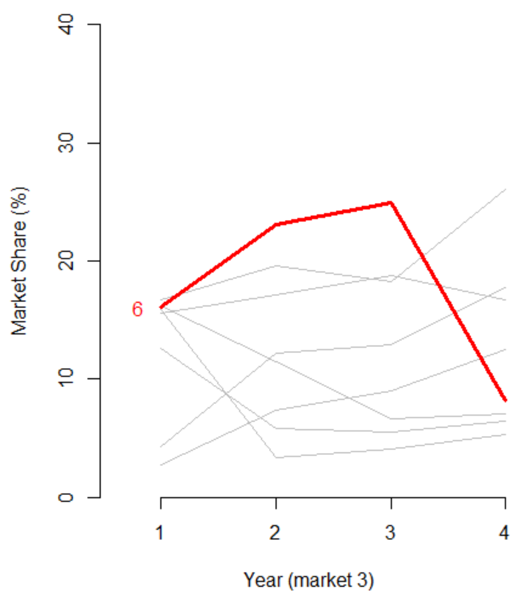
Insurer 6 (market 3)

Team of two actuaries (degrees in Engineering and Physics), in Vancouver, Canada. Used GLMs (Tweedie), no territorial classification, no use of information about other competitors

“*Segments with high market share and low loss ratios were also given some premium increase*”



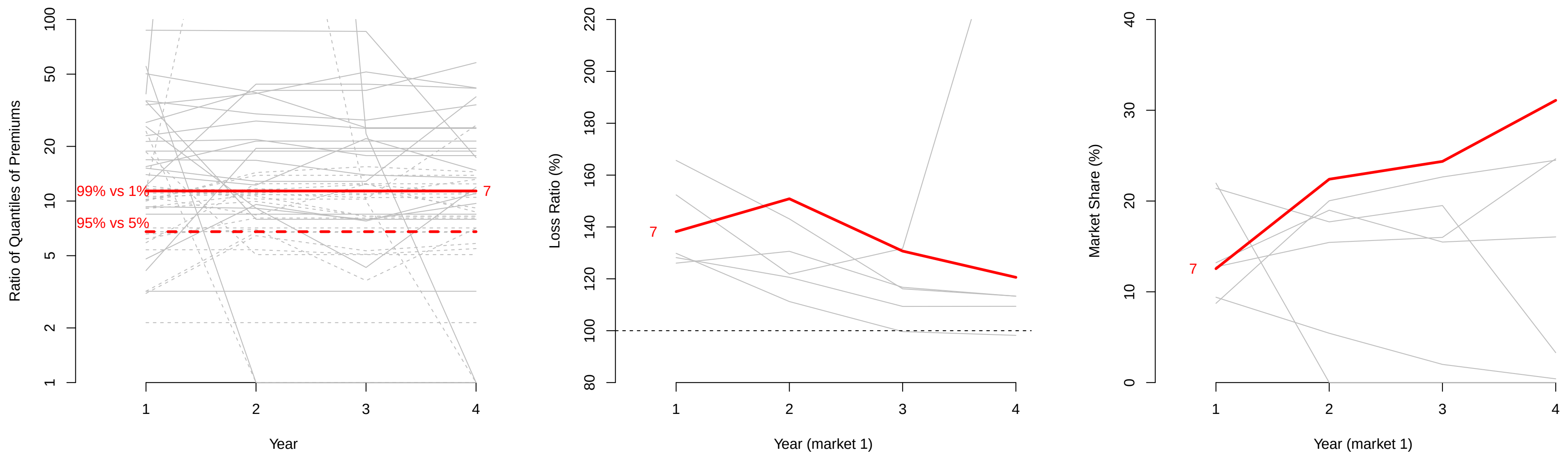




Pricing Game in 2017

Insurer 7 (market 1)

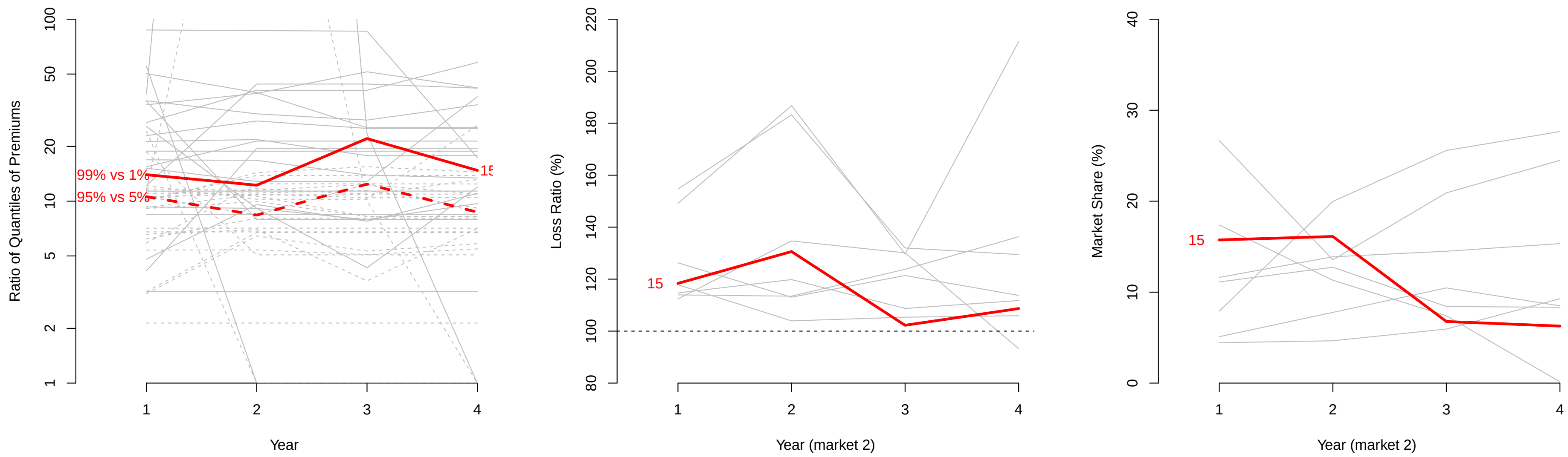
Actuary in France, used random forest for variable selection, and GLMs



Pricing Game in 2017

Insurer 15 (market 2)

Actuary,working as a consultant, Margin Method with iterations, MS Access & MS Excel

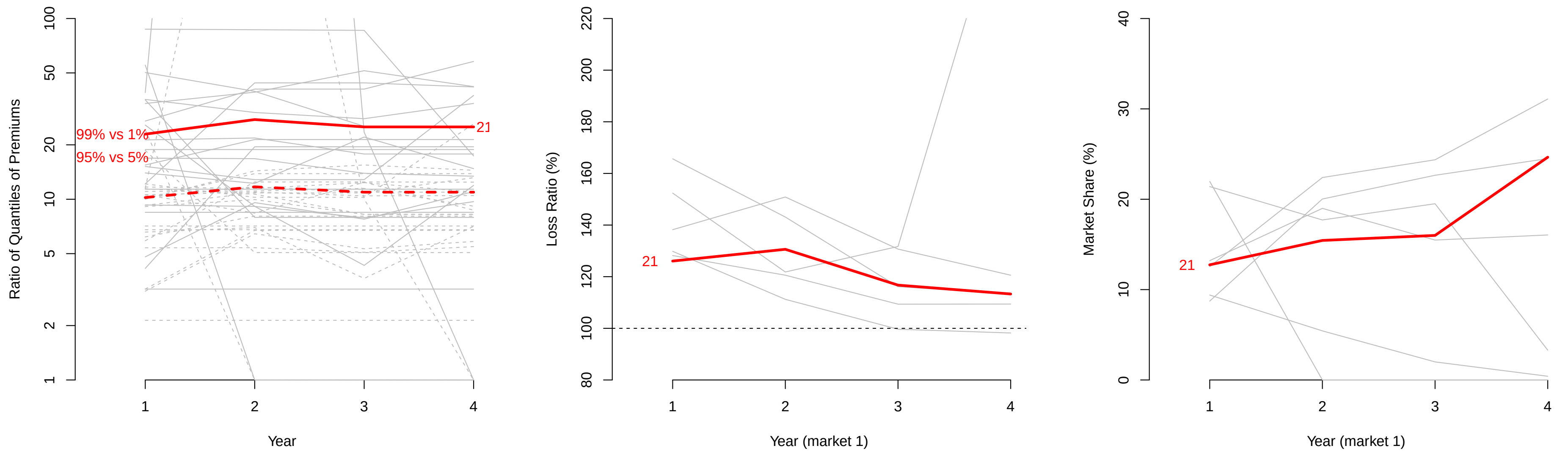


Pricing Game in 2017

Insurer 21 (market 1)

Actuary, working as a consultant, used GLMs, with variable selection using LARS and LASSO

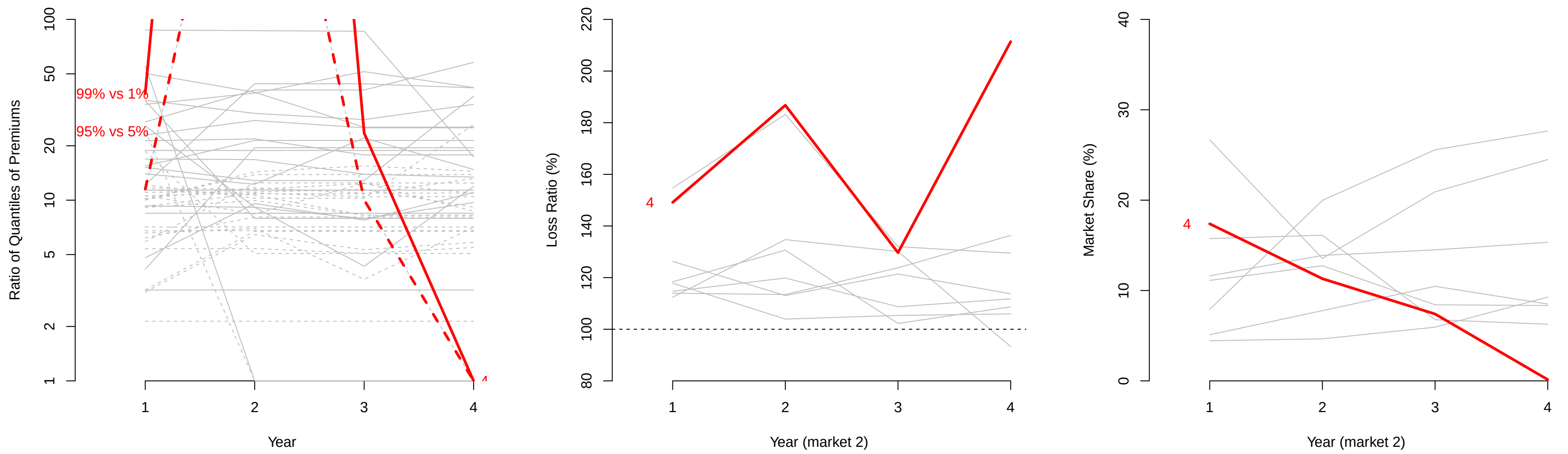
Iterative learning algorithm (codes available on github)



Pricing Game in 2017

Insurer 4 (market 2)

Actuary, working as a consultant,used XGBOOST, used GLMs for year 3.

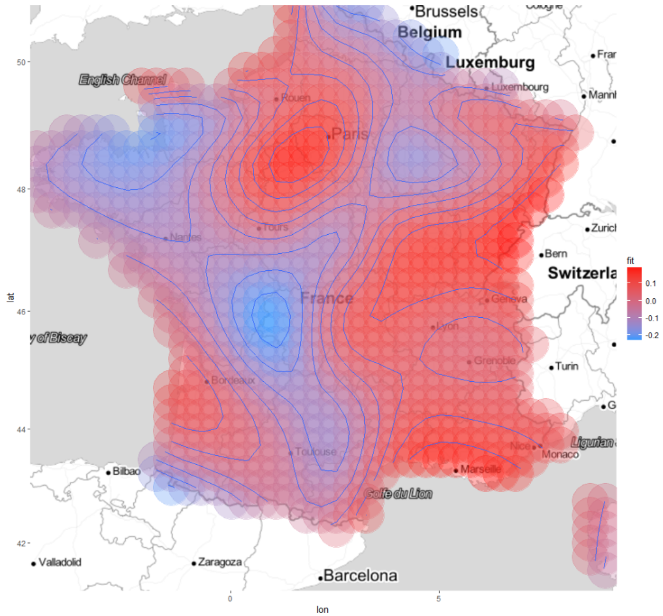


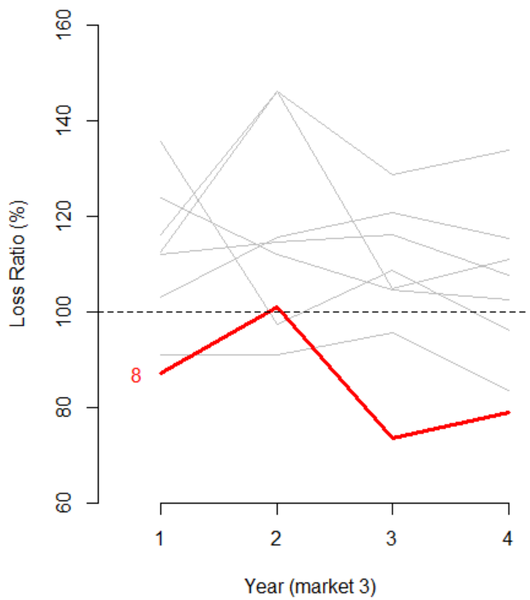
Pricing Game in 2017

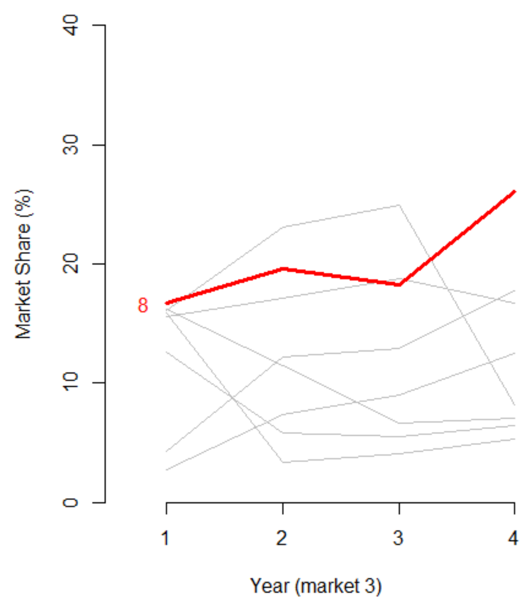
Insurer 8 (market 3)

Mathematician, working on Solvency II software in Austria

Generalized Additive Models with spatial variable







What’s Next ?

- hard to derive theoretical properties of model competition equilibrium
- more on-going field studies... but hard to get players...
- 2018 game, more (too) complicated design...
- 2019 game(s)...

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to be continued...