Pricing Options on Life Insurance

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Introduction

❂ We examine the profitability of financial option contracts on life insurance. Currently, mainstream insurance firms do not offer this type of product. However, with proper pricing models, such a product can be well-structured, profitable, and realistic for a variety of clientele.

❂ We define the product from the client's perspective as the option when young and healthy to guarantee annual life insurance payment rates of a healthy individual when older, regardless of true health at that future time. Clients must judge that the net present value of the upfront option cost will be outweighed by the reduced life insurance payment rates in the future until their death. Therefore the option is valuable to clients only if they are actually unhealthy when their life insurance policy is effective. The option is priced commensurately with the duration of the client's contract.

❂ We develop a discrete multi-state Markovian mortality model that accounts for the possibilities of remaining healthy, entering poor health, becoming disabled, or dying. Using Monte Carlo simulation for analysis, we then construct a stochastic pay-off function [2] which produces fair value pricing for the option based on the client's age. Modifications to the general framework include different mortality dynamics, stochastic discounting rates [3], and cohort effects.

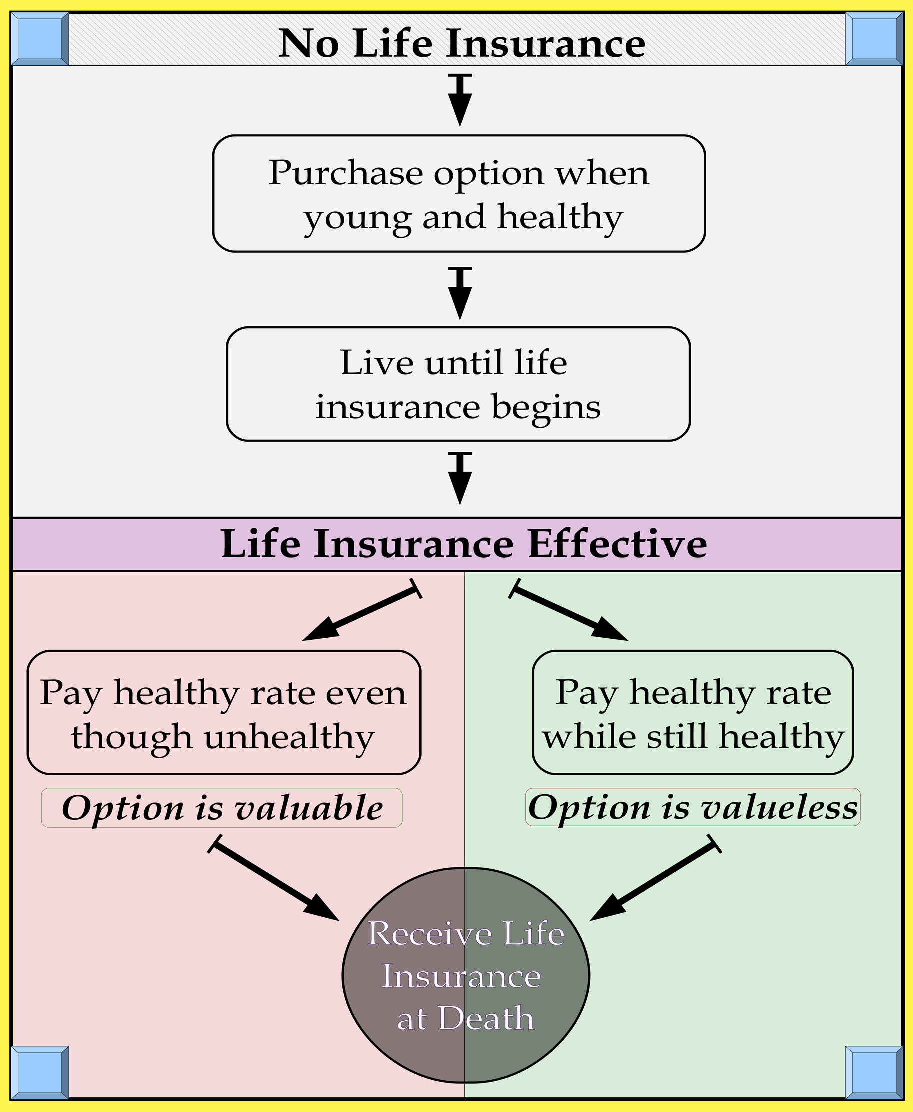
Option Payoff Function

❂ We price the option as the expected value (average) of the option's payoff weighted by the client's probabilities of transitioning into possible health states by age T. The option can only be purchased by clients who are currently healthy.

❂ Option Value $= e^{-\int_t^T r\,ds} \cdot [P_{12}(t,T) \cdot p_2 \cdot (f_2 - f_1) + P_{13}(t,T) \cdot p_3 \cdot (f_3 - f_1)]$

- ✷ t = client's age at purchase of option, $t < T$.
- ✷ r = interest/discount rate for NPV calculation.
- ✷ i = health states (1, 2, 3, or 4, as described below in our model).
- ✷ T = age of client at which the option matures.
- ✷ τ = age of client at death (random varible).
- ✷ $p_i = \mathbb{E}\left[\sum_{s=T}^{\tau} e^{-r(s-T)}\right]$, expected sum of payments from customer to insurance agency until death, if insurance is bought at age T in health state i.
- ✷ $v_i = \mathbb{E}\,[\exp(-r \cdot (\tau - T))]$, present value of expected net claim of \$1.
- ✷ $f_i = v_i / p_i$, fair value of insurance premium starting in state i at age T.
- ✷ $P_{1i}(t,k)$ = probability of being in state i at age T, starting at age t in state 1, computed using the Chapman-Kolmogorov Theorem.
- ✷ 100,000 Monte Carlo simulations was used to calculate expectations for p_i and v_i.

Options on Insurance



Option Values

Interest Rate 5%	
Specialized Matrices	**Option Value at age 25**
Male	\$ 2,180.00
Female	\$ 860.00
Stochastic Interest Rate	\$ 1,250.00
Cohort Effect	\$ 1,510.00
Augmented Male	\$ 2,249.00

✷The above table shows the range of the price of the option at age 25 on a life insurance of \$1,000,000, and how it changes under different life expectancy assumptions. (see example)

✷The cohort effects and the female matrices were adapted from the original. The female matrix was created using SOA mortality rates for females, while the cohort matrix takes into account generational healthcare improvements.

✷The augmented matrix was constructed using the SOA mortality rates as a base case, and interpolating functions to fit transition probabilities which more closely mirror US Census data.

✷Further research should include: improving the reliability and dynamics of the transition matrix, continuous time model as opposed to discrete, alternate option contracts.

Markov Chain

❂ **Transition Matrix**[1]

States	1	2	3	4
1 :Healthy	P_{11}(t)	P_{12}(t)	P_{13}(t)	P_{14}(t)
2 :Poor Health	P_{21}(t)	P_{22}(t)	P_{23}(t)	P_{24}(t)
3 :Disabled	0	0	P_{33}(t)	P_{34}(t)
4 :Deceased	0	0	0	1

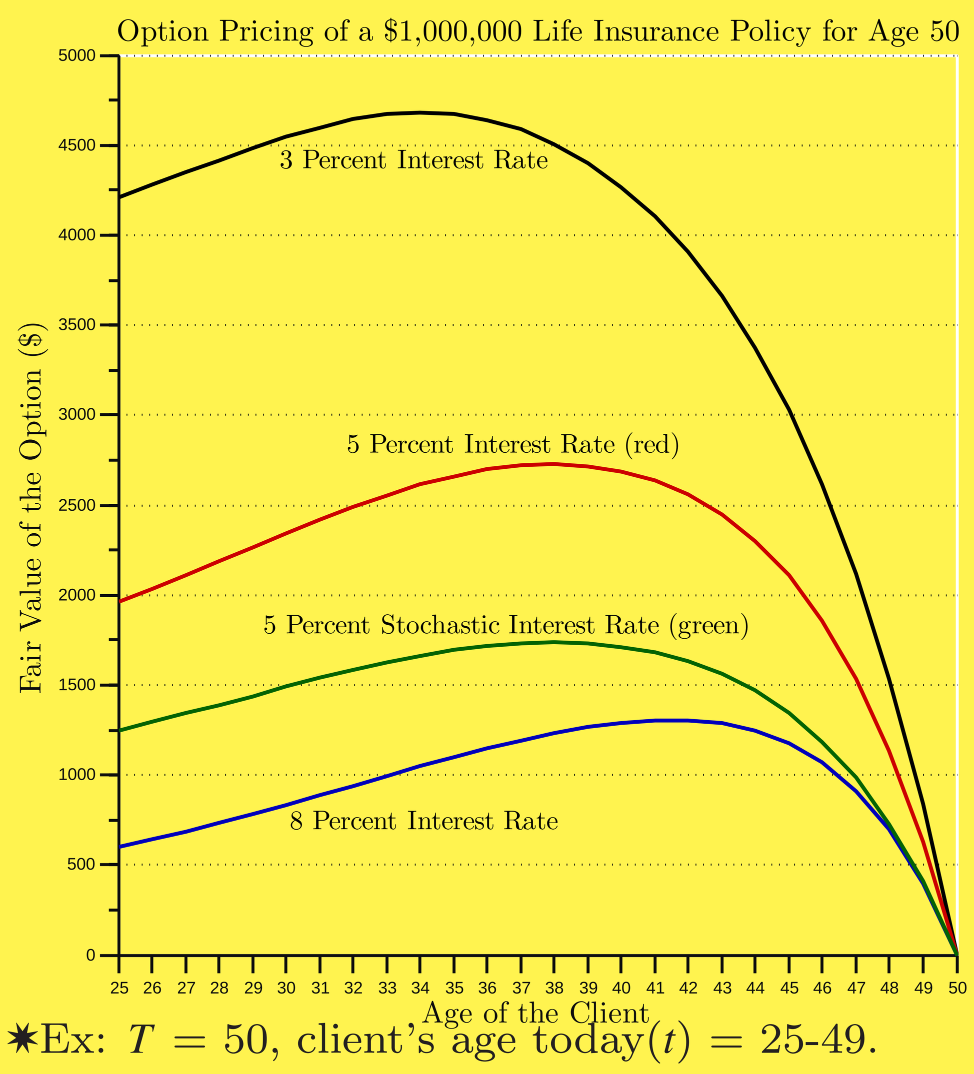
❂ **The Probabilities**

✷ Our matrix above gives the probability of an individual transitioning from one health state to another over one unit of time (1 year).

✷ As an individual ages the probability of transitioning into an unhealthy state increses, while the probability of becoming healthy decreases.

✷The matrix was constructed using mortality rates as published by the Society of Actuaries (SOA)[4].

Example



✷Ex: T = 50, client's age today(t) = 25-49.

✷The curves show that it is cheaper to purchase the option at a young age.

Literature Cited, Software Used, and Acknowledgements

References

[1] Pitacco, E. (1999), "Multistate models for long-term care insurance and related indexing problems", Applied Stochastic Models in Business and Industry, 15: 429–441.

[2] Jones, B. L., "Stochastic Models for Continuing Care Retirement Communities", North American Actuarial Journal.(1997): 50-74.

[3] Ludkovski, M. and Young, V. R. "Indifference Pricing of Annuities and Pure Endowments under Stochastic Hazard and Interest Rates", Insurance: Mathematics and Economics, 42(1): 14–30, 2008.

[4] Society of Actuaries. "RP-2000 Mortality Rates Table", Society of Actuaries, 2000. http://www.soa.org/files/pdf/rp00_mortalitytables.pdf

❂ For this project we used Matlab to run simulations as well as LaTeX for creating this poster.

❂ Finally, a thank you to our faculty advisor, Mike Ludkovski, for his time and expert guidance.