PSTAT 5E
Midterm Key
Winter 2006

Question 1

Let X be the number of jobs a student has

(a) P (a student has 2 jobs) = P (2) = 1 – [P (0) + P (1)] = 1 – [0.5 + 0.3] = 0.2

(b) P (a student has at least 1 job) = P (1) + P (2) = 0.3 + 0.2 = 0.5

(c) The mean number of jobs is just the E (X)

$$E(X) = \sum_{i=1}^{3} p_i x_i = \mu = 0(0.5) + 1(0.3) + 2(0.2) = 0.7$$

Recall that the standard deviation is $\sqrt{Var(X)}$. So we must first find $Var(X)$.

$$Var(X) = \sum_{i=1}^{3} p_i (x_i - \mu)^2 = 0.5(0-0.7)^2 + 0.3(1-0.7)^2 + 0.2(2-0.7)^2 =$$

$$=0.245 + 0.027 + 0.338 = 0.61$$

So, the standard deviation = $\sqrt{Var(X)} = \sqrt{0.61} = 0.781$

Question 2

(a) Let A = {odd number}, then A = {1, 3, 5}.

$$P(A) = P(1) + P(3) + P(5) = 3(\frac{1}{6}) = \frac{1}{2}$$

(b) The sum of the scores is a multiple of 5 if it is equal to 5 or 10. The outcomes from throwing two dices are:

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

The circled outcomes correspond to the cases where the sum is a multiple of 5. Therefore, since the number of circled outcomes is 7 and the total number of outcomes is 36, P (sum is multiple of 5) = 7/36.

(c) Exactly 3 tosses required to get a Head, i.e., we need to have a Tail on the first toss, a Tail on the second toss, and a Head on the third toss:

P (3 tosses required for a Head)
= P (Tail *and* Tail *and* Head)
= P (Tail) P (tail) P (Head)
$= (\frac{1}{2})^3 = \frac{1}{8}$,

where the second equality holds since the tosses are independent.

(d) Neither of two throws results in a '4' means that we get a number different from 4 on the first throw and a number different from 4 on the second throw:

P (neither of two throws results is a '4')
= P (not a '4' on the first throw *and* not a '4' on the second throw)
= P (no '4' on the first throw) P (no '4' on the second throw)
$= (\frac{5}{6})^2 = \frac{25}{36}$,

where the second equality holds because throws are independent.

Question 3

Define the events: T = {household has a TV}
M = {household has a microwave}
TM = {household has both a TV and a microwave}

Then, we are given the probabilities: $P(T) = 0.8$
$P(M) = 0.3$
$P(MT) = 0.2$

(a) The question can be interpreted two ways (both were accepted as correct).

First way: Either T or M (*not both*)
P (either T or M) = $[P(T) - P(TM)] + [P(M) - P(TM)]$
$= [0.8 - 0.2] + [0.3 - 0.2] = 0.7$

Second way: Either T or M *or both*
$P(T \cup M) = P(T) + P(M) - P(TM) = 0.8 + 0.3 - 0.2 = 0.9$

(b) P (neither T nor M) $= 1 - P(T \cup M) = 1 - 0.9 = 0.1$

(c) $P(T \mid M) = \frac{P(TM)}{P(M)} = \frac{0.2}{0.3} = 0.67$

(d) The easiest way to check whether T is independent of M is to use the result in part (c) to check whether $P(T \mid M) = P(T)$
We have: $P(T \mid M) = 0.67 \neq 0.8 = P(T)$.
Therefore, T is **not** independent of M

Question 4
Let X = number of CEOs aware of the information superhighway. This is a binomial experiment with a probability of success $p = P$ *(a CEO is aware)* $= 0.5$

(a) Number of trials: $n = 10$

$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1)$
= 1- 0.011 (from the Binomial table)
= 0.989

(b) $P(X < 5) = P(X \leq 4) = 0.377$ (from the Binomial table)

(c) n = 100, p = 0.5
$E(X) = np = (100)(0.5) = 50$

Question 5
(a) Let X = ounces per cup. Then X has a normal distribution with mean μ (that we don't know) and a standard deviation 0.5, i.e. X ~ N (μ , 0.5)

We need to find μ such that $P(X > 8) = 0.03$

Let's standardize the probability above: $P\left(\frac{X-\mu}{\sigma} > \frac{8-\mu}{0.5}\right) = 0.03$

Then, if we let $z^* = \frac{8-\mu}{0.5}$, we have $P(Z > Z^*) = 0.03$

Recall that Z has the standard normal distribution: N (0,1).

From the Normal table, the area of 0.47 corresponds to a cut-off value of 1.88, i.e., $Z^*=1.88$

Therefore, we have: $\frac{8-\mu}{0.5}$ =1.88, which we solve for μ to get:

$$\mu = 8 - (1.88)\ (0.5) = 7.06$$

(b) Let Y be the number of cups that overflow
Then, this is a binomial experiment (cups either overflow or do not)
Let's define "success" as "a cup overflows"
Then, $p=0.03$ and $n=3$ (Tom, Dick, Harry each get a drink)

We want to find $P(Y < 3 \mid Y \geq 1)$, where $Y < 3$ denotes that not all cups overflow and $Y \geq 1$ that at least one cup overflows.

Applying the formula for conditional probability,

$$P(y<3 \mid y\geq 1) = \frac{P(y < 3 \cap y \geq 1)}{P(y \geq 1)} = \frac{P(y = 1 \cup y = 2)}{P(y \geq 1)} = \frac{P(y = 1) + P(y = 2)}{P(y \geq 1)}$$

Recalling the formula for binomial probability,

$$P\ (Y = 1) = \binom{3}{1}(0.03)^1(1-0.03)^{3-1} = 3(0.03)\ (0.9409) = 0.0847$$

$$P\ (Y = 2) = \binom{3}{2}(0.03)^2(1-0.03)^{3-2} = 3(0.0009)\ (0.97) = 0.00262$$

$$P\ (Y \geq 1) = 1 - P\ (Y < 1) = 1 - P\ (Y = 0) = 1 - \binom{3}{0}(0.03)^0(1-0.03)^3 = 0.08733$$

Therefore, $P\ (Y < 3 \mid Y \geq 1) = \dfrac{0.0847 + 0.00262}{0.08733} = 0.99966$