Question 1
Let $X$ be the number of jobs a student has
(a) $P$ (a student has 2 jobs) = $P(2) = 1 - [P(0) + P(1)] = 1 - [0.5 + 0.3] = 0.2$

(b) $P$ (a student has at least 1 job) = $P(1) + P(2) = 0.3 + 0.2 = 0.5$

(c) The mean number of jobs is just the $E(X)$
$$E(X) = \sum_{i=1}^{3} p_i x_i = \mu = 0(0.5) + 1(0.3) + 2(0.2) = 0.7$$

Recall that the standard deviation is $\sqrt{Var(X)}$. So we must first find $Var(X)$.
$$Var(X) = \sum_{i=1}^{3} p_i(x_i - \mu)^2 = 0.5(0 - 0.7)^2 + 0.3(1 - 0.7)^2 + 0.2(2 - 0.7)^2 =$$
$$= 0.245 + 0.027 + 0.338 = 0.61$$

So, the standard deviation = $\sqrt{Var(X)} = \sqrt{0.61} = 0.781$

Question 2
(a) Let $A = \{\text{odd number}\}$, then $A = \{1, 3, 5\}$.
$$P(A) = P(1) + P(3) + P(5) = 3 \left(\frac{1}{6}\right) = \frac{1}{2}$$

(b) The sum of the scores is a multiple of 5 if it is equal to 5 or 10. The outcomes from throwing two dices are:

```
1,1  2,1  3,1  4,1  5,1  6,1
1,2  2,2  3,2  4,2  5,2  6,2
1,3  2,3  3,3  4,3  5,3  6,3
1,4  2,4  3,4  4,4  5,4  6,4
1,5  2,5  3,5  4,5  5,5  6,5
1,6  2,6  3,6  4,6  5,6  6,6
```

The circled outcomes correspond to the cases where the sum is a multiple of 5. Therefore, since the number of circled outcomes is 7 and the total number of outcomes is 36, $P$ (sum is multiple of 5) = $7/36$. 
(c) Exactly 3 tosses required to get a Head, i.e., we need to have a Tail on the first toss, a Tail on the second toss, and a Head on the third toss:

\[
P (3 \text{ tosses required for a Head}) \\
= P (\text{Tail and Tail and Head}) \\
= P (\text{Tail}) P (\text{tail}) P (\text{Head}) \\
= \left(\frac{1}{2}\right)^3 = \frac{1}{8},
\]

where the second equality holds since the tosses are independent.

(d) Neither of two throws results in a ‘4’ means that we get a number different from 4 on the first throw and a number different from 4 on the second throw:

\[
P (\text{neither of two throws results is a ‘4’}) \\
= P (\text{not a ‘4’ on the first throw and not a ‘4’ on the second throw}) \\
= P (\text{no ‘4’ on the first throw}) P (\text{no ‘4’ on the second throw}) \\
= \left(\frac{5}{6}\right)^2 = \frac{25}{36},
\]

where the second equality holds because throws are independent.

**Question 3**

Define the events:

- \( T = \{ \text{household has a TV} \} \)
- \( M = \{ \text{household has a microwave} \} \)
- \( TM = \{ \text{household has both a TV and a microwave} \} \)

Then, we are given the probabilities:

\[
P (T) = 0.8 \\
P (M) = 0.3 \\
P (MT) = 0.2
\]

(a) The question can be interpreted two ways (both were accepted as correct).

**First way:** Either \( T \) or \( M \) (not both)

\[
P (\text{either } T \text{ or } M) = [P (T) - P (TM)] + [P (M) - P (TM)] \\
= [0.8 - 0.2] + [0.3 - 0.2] = 0.7
\]

**Second way:** Either \( T \) or \( M \) or both

\[
P (T \cup M) = P (T) + P (M) - P (TM) = 0.8 + 0.3 - 0.2 = 0.9
\]

(b) \( P (\text{neither } T \text{ nor } M) = 1 - P (T \cup M) = 1 - 0.9 = 0.1
\]

(c) \( P (T | M) = \frac{P(TM)}{P(M)} = \frac{0.2}{0.3} = 0.67 \)
(d) The easiest way to check whether $T$ is independent of $M$ is to use the result in part (c) to check whether $P(T|M) = P(T)$

We have: $P(T|M) = 0.67 \neq 0.8 = P(T)$.

Therefore, $T$ is not independent of $M$.

**Question 4**

Let $X =$ number of CEOs aware of the information superhighway. This is a binomial experiment with a probability of success $p = P(a \text{ CEO is aware}) = 0.5$

(a) Number of trials: $n = 10$

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1)$$

= $1 - 0.011$ (from the Binomial table)

= $0.989$

(b) $P(X < 5) = P(X \leq 4) = 0.377$ (from the Binomial table)

(c) $n = 100$, $p = 0.5$

$E(X) = np = (100)(0.5) = 50$

**Question 5**

(a) Let $X =$ ounces per cup. Then $X$ has a normal distribution with mean $\mu$ (that we don’t know) and a standard deviation 0.5, i.e. $X \sim N(\mu, 0.5)$

We need to find $\mu$ such that $P(X > 8) = 0.03$

Let’s standardize the probability above: $P\left(\frac{X - \mu}{\sigma} > \frac{8 - \mu}{0.5}\right) = 0.03$

Then, if we let $z^* = \frac{8 - \mu}{0.5}$, we have $P(Z > Z^*) = 0.03$

Recall that $Z$ has the standard normal distribution: $N(0,1)$.

From the Normal table, the area of 0.47 corresponds to a cut-off value of 1.88, i.e., $Z^* = 1.88$

Therefore, we have: $\frac{8 - \mu}{0.5} = 1.88$, which we solve for $\mu$ to get:

$$\mu = 8 - (1.88)(0.5) = 7.06$$
(b) Let $Y$ be the number of cups that overflow
Then, this is a binomial experiment (cups either overflow or do not)
Let’s define “success” as “a cup overflows”
Then, $p=0.03$ and $n=3$ (Tom, Dick, Harry each get a drink)

We want to find $P(Y < 3 \mid Y \geq 1)$, where $Y < 3$ denotes that not all cups overflow and $Y \geq 1$ that at least one cup overflows.

Applying the formula for conditional probability,

$$P(y < 3 \mid y \geq 1) = \frac{P(y < 3 \cap y \geq 1)}{P(y \geq 1)} = \frac{P(y = 1 \cup y = 2)}{P(y \geq 1)} = \frac{P(y = 1) + P(y = 2)}{P(y \geq 1)}$$

Recalling the formula for binomial probability,

$$P (Y = 1) = \binom{3}{1}(0.03)^1(1 - 0.03)^{3-1} = 3(0.03)(0.9409) = 0.0847$$

$$P (Y = 2) = \binom{3}{2}(0.03)^2(1 - 0.03)^{3-2} = 3(0.0009)(0.97) = 0.00262$$

$$P (Y \geq 1) = 1 - P (Y < 1) = 1 - P (Y = 0) = 1 - \binom{3}{0}(0.03)^0(1 - 0.03)^3 = 0.08733$$

Therefore, $P (Y < 3 \mid Y \geq 1) = \frac{0.0847 + 0.00262}{0.08733} = 0.99966$