6.13.  
\textbf{a.} The population from which we are randomly sampling \( n = 35 \) measurements is not necessarily normally distributed. However, the sampling distribution of \( \bar{x} \) does have an approximately normal distribution, with mean \( \mu \) and standard deviation \( \sigma / \sqrt{n} \). The probability of interest is \( P[|\bar{x} - \mu| < 1] = P[-1 < (\bar{x} - \mu) < 1] = P[(\bar{x} - \mu) < 1] - P[(\bar{x} - \mu) < -1] \)

Standardizing, we have:
\[
P[(\bar{x} - \mu) < 1] - P[(\bar{x} - \mu) < -1] = P[\bar{x} < 1 + \mu] - P[\bar{x} < 1 - \mu] = P \left[ z < \frac{(1 + \mu) - \mu}{\sigma / \sqrt{n}} \right] - P \left[ z < \frac{(-1 + \mu) - \mu}{\sigma / \sqrt{n}} \right]
\]
\[
= P \left[ z < \frac{1}{\sigma / \sqrt{n}} \right] - P \left[ z < \frac{-1}{\sigma / \sqrt{n}} \right] = 2 \left( \frac{0.1879}{12} \right) = 0.3758
\]

where, since \( \sigma \) is unknown, we use the sample standard deviation \( s = 12 \) (we can do this, since the sample size is large enough).

\textbf{b.} No. There are many possible values for \( x \), the actual percent tax savings, as given by the probability distribution for \( x \).

6.16.  
\textbf{a.} The sample mean \( \bar{x} \) is normally distributed, since the original strength measurements are Normally distributed, with mean \( \mu \) and standard deviation \( \sigma / \sqrt{n} = 2 / \sqrt{10} \).

\textbf{b.} If \( \mu = 21 \) and \( n = 10 \), the \( z \)-value corresponding to \( \bar{x} = 20 \) is
\[
z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{20 - 21}{2 / \sqrt{10}} = -1.58 \text{ and}
\]
\[
P[\bar{x} < 20] = P[z < -1.58] = .5 - .4429 = .0571
\]

\textbf{c.} Refer to part b and assume that \( \mu \) is unknown. It is necessary to find a value \( \mu^{*} \) so that \( P[\bar{x} < 20] = .001 \)

Recall that if \( \mu = \mu^{*} \), the \( z \)-value corresponding to \( \bar{x} = 20 \) is
\[
z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{20 - \mu^{*}}{2 / \sqrt{10}} = \frac{20 - \mu^{*}}{.632}
\]

It is necessary then to have
\[ P[\bar{x} < 20] = P[z < \frac{20 - \mu^*}{.632}] = .5 - A\left(\frac{20 - \mu^*}{.632}\right) = .001 \]

or \[ A\left(\frac{20 - \mu^*}{.632}\right) = .4990 \]

Refer to Figure 6.3. The value of \( z \) that cuts off .001 in the left-hand tail of the normally distribution is negative and is such that \( A(z_0) = .499 \). From Table 3, this value is \( z_0 = -3.08 \).

Solving for \( z_0 = \frac{20 - \mu^*}{.632} = -3.08 \) or \( \mu^* = 21.948 \).

6.25.

It is given that \( n = 1600 \) and \( \hat{p} = .71 \), where \( p \) is the proportion of “upscale” men in the population who find shopping by computer convenient.

a. The sample proportion \( \hat{p} \) is approximately normally distributed with mean \( p \) (unknown) and standard deviation \( \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.71(.29)}{1600}} = .011 \), where we use \( \hat{p} \) to approximate \( p \).

b. \[ P[.03 \leq (\hat{p} - p) \leq .03] = P[.03 + p < \hat{p} < .03 + p] = P\left[\frac{(.03 + p) - \hat{p}}{.011344} < z < \frac{(.03 + p) - \hat{p}}{.011344}\right] = \]

\[ = P[-.03 / .011344 < z < .03 / .011344] = P[-2.64 < z < 2.64] = 2(.4959) = .9918 \]
6.27. It is given that \( n = 300 \) and \( p = .8 \), where \( p \) is the proportion of married people who feel they have an equal say in major home purchases.

a. The sample proportion \( \hat{p} \) is approximately normally distributed with mean \( p = .8 \) and standard deviation \( \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.8(.2)}{300}} = .02309 \)

Then,
\[
P \left( \hat{p} > .85 \right) = P \left( z > \frac{\hat{p} - p}{\sigma_{\hat{p}}} \right) = P \left( z > \frac{.85 - .8}{.02309} \right) = P \left( z > 2.17 \right) = .5 - .4850 = .0150
\]

b. Since \( \hat{p} \) is approximately normally distributed, we would expect approximately 95% of the sample proportions to lie within two standard deviations of the mean. That is,
\[
p \pm 2\sigma_{\hat{p}} \Rightarrow .8 \pm 2(.02309) \Rightarrow .8 \pm .046 \text{ or } .754 \text{ to } .846
\]

c. 
\[
P \left( |\hat{p} - p| > 0.05 \right) = 1 - P \left( |\hat{p} - p| < 0.05 \right) = 1 - P \left( 0.75 < \hat{p} < 0.85 \right) = 1 - P \left( -2.17 < z < 2.17 \right) = 2(0.485) = 0.97
\]

6.42.

a-b. It is given that \( \mu = 31,256, n = 25 \) and \( \sigma = 1550 \). The sample mean \( \bar{x} \) is approximately normally distributed, according to the Central Limit Theorem, with \( \mu = 31,256 \) and standard deviation
\[
\sigma / \sqrt{n} = 1550 / \sqrt{25} = 310
\]

c. \( P \left( \bar{x} > 32,000 \right) = P \left( z > \frac{32,000 - 31,256}{310} \right) = P \left( z > 2.40 \right) = .5 - .4918 = .0082 \)
and \( P \left( \bar{x} > 33,000 \right) = P \left( z > \frac{33,000 - 31,256}{310} \right) = P \left( z > 5.63 \right) = 0 \)

d. Since \( \bar{x} \) is approximately normally distributed, approximately 95% of the values of \( \bar{x} \) should lie within 1.96 (or 2) standard deviations of the mean \( \mu = 31,256 \). That is, the sample mean \( \bar{x} \) should lie with 95% probability between
\[
\mu \pm 2\frac{\sigma}{\sqrt{n}} \Rightarrow 31,256 \pm 2\frac{1550}{\sqrt{25}} \Rightarrow 31,256 \pm 620
\]
or $30,636 to $31,876.
6.43.
It is given that \( n = 1000 \) and \( p = .86 \) where \( p \) is the proportion of freshmen receiving financial aid from parents or family.

a. The sample proportion \( \hat{p} \) is approximately normally distributed with mean \( \mu = .86 \) and standard deviation \( \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.86(.14)}{1000}} = .01097 \)

\[ \hat{p} = \frac{\sqrt{pq}}{n} = \sqrt{\frac{.86(.14)}{1000}} = .01097 \]

b. \( P[-.02 < \hat{p} - p < .02] = P[-.02 / .01097 < z < .02 / .01097] \)
\[ = P[-1.82 < z < 1.82] = 2(.4656) = .9312 \]

c. \( P[ \hat{p} > .9] = P[z > \frac{\hat{p} - p}{\sigma_{\hat{p}}}] = P[z > \frac{.9 - .86}{.01097}] = P[z > 3.65] = 0 \)
This is not a likely event, and we would not expect to see a sample proportion in this range.