PSTAT 5E

Practice Homework 5
Solutions

6.1.
Regardless of the shape of the population from which we are sampling, the sampling distribution of the sample mean will have a mean μ equal to the mean of the population from which we are sampling, and a standard deviation equal to $\sigma / \sqrt{n}$.

a. $E(\bar{X}) = \mu = 10$; $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 3/\sqrt{25} = .6$
b. $E(\bar{X}) = \mu = 5$; $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 2/\sqrt{100} = .2$
c. $E(\bar{X}) = \mu = 120$; $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 1/\sqrt{6} = .4082$

6.2.
a. If the population from which the sample is drawn is normal, the distribution of $\bar{X}$ is also normal for **<u>any</u>** value of n.

b. If the population from which the sample is drawn is not normal, The Central Limit Theorem tells us that for sample sizes greater than about $n = 25$, the sampling distribution of $\bar{X}$ will be approximately Normal. Hence, we can be relatively certain that the sampling distribution of $\bar{X}$ for parts **a** and **b** will be approximately Normal. However, the sample size in part **c**, $n = 6$, is too small to assume that the distribution of $\bar{X}$ is approximately Normal (i.e., we cannot apply the Central Limit Theorem in this case).

6.15.
Let X = # of heavy-duty trailers per hour. X is approximately Normally distributed with $\mu = 50$ and $\sigma = 7$. Then the sampling distribution of $\bar{X}$ has an approximately normal distribution, with mean $\mu = 50$ and standard deviation $\sigma / \sqrt{n} = 7 / \sqrt{25} = 1.4$.

a. The probability of interests is $P[\bar{X} > 55]$.

The Z-score is $z = \dfrac{\bar{x} - u}{\sigma / \sqrt{n}} = \dfrac{55 - 50}{1.4} = 3.57$ and $P[\bar{X} > 55] = P[Z > 3.57] = .5 - A(3.57) \approx .5 - .5 = 0$

b. This is similar to part **a**, except that $\bar{X}$ is calculated based on $n = 4$ observations.

The Z-score is $z = \dfrac{\bar{x} - u}{\sigma / \sqrt{n}} = \dfrac{55 - 50}{7 / \sqrt{4}} = 1.43$ and $P[\bar{X} > 55] = P[Z > 1.43] = .5 - A(1.43) = .5 - .4236 = .0764$

c. The total number of trucks for a four-hour period is the sum of four observations, $\sum_{i=1}^{4} x_i$, each of which is Normally distributed with μ = 50 and σ =7. The sum will also be Normally distributed with mean $n\mu$ = 4(50) = 200 and standard deviation $\sqrt{n}\sigma = (\sqrt{4})(7) = 14$.
Hence, the Z-value corresponding to $\sum x_i$ = 180 is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{180 - 200}{14} = -1.43$$

and $P[\sum x_i > 180]$ = P[Z > -1.43] = .5 + .4236 = .9236

6.18.

a. $\mu_{\hat{p}} = p = .3; \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.3(.7)}{100}} = .0458$

b. $\mu_{\hat{p}} = p = .1; \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.1(.9)}{400}} = .015$

c. $\mu_{\hat{p}} = p = .6; \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.6(.4)}{250}} = .0310$

6.24.

For n = 400 and p=.8, the sample proportion $\hat{p}$ is approximately normally distributed with

$\mu_{\hat{p}} = p = .8; \ \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.8(.2)}{400}} = .02$

a. $P[\hat{p} > .83] = P[Z > \frac{.83 - .8}{.02}] = P[Z > 1.5] = .5 - .4332 = .0668$

b. $P[.76 < \hat{p} < .84] = P[\frac{.76 - .8}{.02} < Z < \frac{.84 - .8}{.02}] = P[-2 < Z < 2] = 2(.4772) = .9544$

6.26.
It is given that n = 250 and p is the proportion of people whose home costs more than the median home price, $112,000. Hence (using the definition of the median), p = .5.

a. The sample proportion $\hat{p}$ is approximately Normally distributed with mean p = .5 and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.5(.5)}{250}} = .03162$

b. $P[\hat{p} > .66] = P[Z > \frac{\hat{p} - p}{\sigma_{\hat{p}}}] = P[Z > \frac{.66 - .5}{.03162}] = P[Z > 5.06] \approx 0$

c. If a sample of size n = 250 produces a sample proportion of $\hat{p} = \frac{165}{250}$ =.66, this would be a very

unlikely occurrence under the assumption that p = .5. Either we have observed a very unlikely event, or perhaps (1) the sample was not randomly selected from all home buyers in the first quarter of 1994, or (2) the median home price nationally may be incorrect.

6.39.

a. For n = 250, the sample proportion $\hat{p}$ is approximately Normally distributed with mean p (unknown) and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.25(.75)}{250}} = .02739$

b. $P\,[(\hat{p} - p) > 0.01 \text{ or } (\hat{p} - p) < -0.01] = 1 - P\,[-0.01 < (\hat{p} - p) < 0.01]$

$$= 1 - P\,[-0.01 + p < \hat{p} < 0.01 + p]$$

$$= 1 - P\left[\frac{(-0.01 + p) - p}{0.027} < \frac{\hat{p} - p}{\sigma_{\hat{p}}} < \frac{(0.01 + p) - p}{0.027}\right]$$

$$= 1 - P\,[-0.37 < Z < 0.37] = 1 - 2(.1443) = 0.7114$$

6.41.

a. It is given that μ = 1085. If the range R=1135 – 1005 = 130 represents three standard deviations on either side of the mean (or 6σ), then 6σ =130 or σ =21.67. The sample mean $\overline{X}$ is approximately Normally distributed, according to the Central Limit Theorem, with mean μ = 1085 and standard deviation $\sigma / \sqrt{n} \approx 21.67 / \sqrt{100} = 2.167$

b. $P\,[\overline{X} > 1090] = P\,[Z > \frac{1090 - 1085}{2.167}] = P\,[Z > 2.31] = .5 - .4896 = .0104$

c. $P\,[\overline{X} < 1078] = P\,[Z < \frac{1078 - 1085}{2.167}] = P\,[Z > -3.23] \approx 0$

If the observed sample mean is $\overline{x}$ = 1078, a very unlikely occurrence, we might conclude that one of our original assumptions, either about the mean, the standard deviation, or perhaps the randomness of our sample should be questioned.