1a) Consider a European put option with strike price 105 and time-to-expiry 2 months. The current underlying stock price is 100 and the risk-free rate is 5%. When the stock rises, the growth rate is $u = 1.3$, and when the stock falls, the rate is $d = 0.7$. Use a binomial tree with a step size of one month to price this option. Carefully draw and label the tree, naming the nodes, writing stock prices above each node, and option prices below each node. Use 1 month = 1/12 years. (6pts)

\[
p = \frac{e^{0.05 \times \frac{1}{12}} - 0.7}{1.3 - 0.7} = 0.50676
\]

\[
f_{uu} = 0 \quad f_{ud} = 14 \quad f_{dd} = 56
\]

@B, \( f_u = e^{-0.05 \times \frac{1}{12}} \left[ p \cdot f_{uu} + (1-p) \cdot f_{ud} \right] = 6.8739 \)

@C, \( f_d = e^{-0.05 \times \frac{1}{12}} \left[ p \cdot f_{ud} + (1-p) \cdot f_{dd} \right] = 34.5634 \)

@A, \( f = e^{-0.05 \times \frac{1}{12}} \left[ p \cdot f_u + (1-p) \cdot f_d \right] = 20.4406 \)

1b) Repeat the exercise in part a, but now the option is an American put option. (6pts)

@B, early exercise is negative (\(-25\))

\( f_u = 6.8739 \)

\( f^* = 6.8739 \quad \text{No exercise} \)

@C, payoff from early exercise = 105 - 70 = 35

\( f_d = 34.5634 \)

\( \text{Max (35, } f_d) = 35 \)

\( f^*_d = 35 \quad \text{Early exercise is optimal} \)

@A, payoff from early exercise = 5

\( f = e^{-0.05 \times \frac{1}{12}} \left[ p \cdot f_u + (1-p) \cdot f^*_d \right] = 20.655 \)

\( \text{Max (5, } f) = 20.655 \)

\( f^*_d = 20.655 \quad \text{No early exercise} \)
2) (2pts) The lower bound on the price of a European call option on a non-dividend paying stock is

a) $S_0$  

b) $K$  

c) $S_0 - Ke^{-rT}$  

d) $Ke^{-rT} - S_0$

3) (2pts) Consider an American put option on a non-dividend paying stock. All else equal, if the risk-free rate increases, the price of the option will (circle one)

a) increase  

b) decrease  

c) remain the same  

d) cannot be determined

4) (2pts) Consider a European call option on a dividend paying stock. All else equal, if the volatility increases, the price of the option will (circle one)

a) increase  

b) decrease  

c) remain the same  

d) cannot be determined

5) (2pts) Consider an American call option on a dividend-paying stock. It may be optimal to exercise early

a) before the dividend is paid out  

b) after the dividend is paid out  

c) if you believe the stock will go down  

d) it is never optimal to exercise this option early

Tell me a joke (keep it clean)