SCORING BREAKDOWN

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FINAL EXAMINATION

Instructions: Show all work. Round answers to nearest penny. Box all final answers.

For full credit on profit diagrams, draw individual positions (using dashed curve), and combined (net) positions (using solid curve). Indicate all relevant prices (for asset and option), strike prices.

Throughout this exam, the continuously compounded risk-free rate is 2.00% per year. 1 month = 1/12 year.

1a) (15pts) A down-and-out call is a call option that ceases to exist for its remaining lifetime (has value zero) if the asset price reaches or falls below a certain barrier level H. Consider a European-style down-and-out call with barrier H = 95, strike price 105, and time-to-expiry 3 months. The current underlying stock price is 100, and the stock’s volatility is 30%. Use a binomial tree with a step size of one month to price this option. Carefully draw and label the tree, naming the nodes, writing stock prices above each node, and option prices below each node.

\[
\Delta t = \frac{1}{12}, \\
U = e^{\sigma \sqrt{\Delta t}} = 1.09, \\
D = \frac{1}{U} = 0.92, \\
P = e^{r \Delta t} - D = 0.45
\]

\[
\begin{array}{ccccccc}
& & & & & & \\
+ & 129.67 & & & & & \\
0 & 118.91 & 110.59 & & & & \\
1 & 109.05 & 101.41 & 94.21 & & & \\
2 & 100 & 93 & 86.49 & 80.44 & & \\
3 & & & & & & \\
\end{array}
\]

1b) (4pts) Compared to the price of a call option with same strike and time-to-expiry, the price of the down-and-out call is

i) higher 
ii) lower 
iii) the same 
iv) cannot be determined

2) MULTIPLE CHOICE QUESTIONS. CIRCLE THE CORRECT RESPONSE(S).

2a) (3pts) The explicit finite difference scheme is equivalent to which of the following?

i) pricing on a binomial tree 
ii) pricing on a trinomial tree 
iii) Monte Carlo simulation 
iv) an asymptotic expansion
2b) (3pts) Which method works best for pricing an American call option on a dividend-paying stock?

i) the Black-Scholes-Merton formula
ii) Monte Carlo simulation
iii) a Binomial Tree
iv) Risk-Neutral valuation

2c) (3pts) Which method works best for pricing an American call option on a non-dividend-paying stock?

i) the Black-Scholes-Merton formula
ii) Monte Carlo simulation
iii) a Binomial Tree
iv) Risk-Neutral valuation

2d) (3pts) In order to make a portfolio delta, gamma, and vega neutral, the following instruments are required:

i) the underlying only
ii) the underlying and a traded option
iii) the underlying and two traded options
iv) the underlying and three traded options

2e) (3pts) Which statement(s) are true about Brownian motion?

i) The path of a Brownian motion is continuous
ii) The path of a Brownian motion is continuously differentiable
iii) Brownian motion is a Markov process
iv) The increment of a Brownian motion measured from time 0 to t is normally distributed with mean zero and standard deviation \( \sqrt{t} \)

2f) (3pts) We had a guest speaker give a talk in our course. This guest speaker works as a quant in which financial market?

i) energy
ii) foreign exchange
iii) corporates
iv) credit
v) equities

2g) (1pt) What is the last name of the TA for our course?

i) Shen
ii) Lorig
iii) Tashman
iv) Jones
v) Scholes
3) The Black-Scholes formula for a call option on a non-dividend-paying stock is given below.

\[ c = S_0 N(d_1) - K e^{-rT} N(d_2) \]
\[ d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma \sqrt{T}} \]
\[ d_2 = d_1 - \sigma \sqrt{T} \]

3a) (5pts) Using put-call parity, write the Black-Scholes formula for a put option on this stock. Simplify as much as possible.

\[ p + S_0 = c + Ke^{-rT} \quad \text{(Put-Call Parity)} \]
\[ c = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad \text{(BSM)} \]

\[ p = S_0 [N(d_1) - 1] - Ke^{-rT} [N(d_2) - 1] \]

3b) (5pts) Consider an at-the-money European call option on a non-dividend-paying stock. The option expires in one year, and the volatility is 40%. Compute the Black-Scholes price of this option when \( S_0 = 100 \).

\[ \Delta \text{call} \quad \Delta \text{put} \]

\[ \Delta_c = N(d_1) = 0.5987 \quad \Delta_p = N(d_1) - 1 = 0.4013 \]

3c) (5pts) Consider the same option from part b, and compute its delta. Additionally, consider an at-the-money European put option on the same stock with expiry of one year. Compute the delta of this put option.

\[ d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma \sqrt{T}} = 0.25 + 0.25 = 0.50 \]
\[ d_2 = 0.25 - 0.4 = -0.15 \]

3d) (5pts) Consider a European call option on a non-dividend-paying stock with current price \$50, time-to-expiry 6 months, and strike price \$55. If the market price of the option is \$3.91, the implied volatility is closest to

i) 10%  ii) 20%  iii) 30%  iv) 40%  v) 50%

\[ \Delta_c = N(d_1) \]
\[ N(d_1) = 0.5987 \]
\[ N(d_2) = 0.4013 \]
4) (10pts) XYZ stock is trading at $49 per share. Ann writes a call option on this stock with a delta of 0.52 and a gamma of 0.15. She wishes to make the portfolio delta neutral and gamma neutral. To this end, she considers another call option on the stock which has a delta of 0.80 and a gamma of 0.075. Write the trades which must be done (in addition to the written call) in order to achieve delta and gamma neutrality.

\[ \Delta = -52 + x + 100 \times (0.80) y = 0 \]

\[ (\neq -15 + 100 \times (0.075) y = 0 \implies y = \frac{15}{7.5} = \frac{5}{2.5} \]

Plug \( y \) into \( \Delta \) equ:

\[ -52 + x + 100 \times (0.80) 2 = \]

5a) (8pts) The Ornstein-Uhlenbeck process is a stochastic process \( r_t \) given by the following stochastic differential equation (SDE):

\[ dr_t = \theta (\mu - r_t) dt + \sigma dW_t \]

where \( W_t \) is a standard Brownian motion, and \( \theta, \mu, \) and \( \sigma \) are positive constants.

Consider the function \( f(t, r_t) = r_t e^{\theta t} \)

Apply Itô's lemma to this function to write the SDE for the infinitesimal change in this function, \( df(t, r_t) \)

\[
\begin{align*}
\frac{df_t}{dt} &= \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial r} dr_t + \frac{1}{2} \frac{\partial^2 f}{\partial r^2} dr_t^2 \\
&= \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial r^2} dr_t^2 \\
&= r_t \theta e^{\theta t} dt + e^{\theta t} \left[ \theta (\mu - r_t) dt + \sigma dW_t \right] \\
&= r_t \theta e^{\theta t} dt + e^{\theta t} \left[ \theta \mu dt + \sigma e^{\theta t} dt \right] \\
&= \theta r_t e^{\theta t} dt + \sigma e^{\theta t} dt
\end{align*}
\]

5b) (2pts) Suppose \( \theta = 1, \mu = .03, \) and \( \sigma = 0.4. \) For a very high value of \( r_t \) (say 10%), which is the most likely outcome of the next increment \( dr_t \)

i) it will be positive  ii) it will be negative  iii) it will be zero

Plug values into SDE and look at \( dr_t \)
6) Greg C places the option trades given below, where each option is on stock ABC and has time-to-expiry 3 months.

- Buy one call with strike $55 and price \( c_L \)
- Buy one call with strike $65 and price \( c_H \)
- Sell two calls with strike $60 and price \( c_M \)

6a) (10pts) Carefully draw the profit diagram for this set of trades

\[
\text{Profit} = 5 + 2c_m - c_L - c_H
\]

6b) (5pts) What is the maximum profit for this set of trades (in terms of the variables, of course)?

\[100 \left( 5 + 2c_m - c_L - c_H \right)\]

Under what condition(s) does this occur?

\[S_T = 60\]

6c) (5pts) What is the maximum loss for this set of trades?

\[100 \left| 2c_m - c_L - c_H \right|\]

Under what condition(s) does this occur?

\[S_T \leq 55, \quad S_T \geq 65\]

6d) (2pts) What is the name of this strategy? \textit{long butterfly}

Tell me a joke (keep it clean)

END OF EXAM. I HOPE YOU ENJOYED THIS COURSE. HAVE A GREAT SUMMER!!