MIDTERM EXAMINATION

Instructions: Write your name on each page. Put your ID on page 1. Throughout the exam, the risk-free rate of interest is 2%, this is a continuously compounded annual rate. Use 1 month = 1/12 years. Show all work clearly, write all relevant formulas, and put a box around final answers.

MULTIPLE CHOICE QUESTIONS 1-4

1) (5pts) Consider two American put options on the same dividend-paying stock, with the same expiry. The option with the higher strike is __________ than the option with the lower strike
   a) more valuable   b) less valuable   c) cannot be determined

2) (5pts) An investor holds shares of XYZ and has built a large profit. The company is involved in a lawsuit, and the investor is afraid of losing this profit. Which strategy can help lock in most of the profit?
   a) covered call   b) short strangle   c) long butterfly   d) protective put

3) (5pts) The current price of an option is equal to
   a) the expected payoff under the physical probability measure
   b) the expected payoff under the risk-neutral probability measure
   c) the expected payoff under the physical probability measure, discounted at the risk-free rate
   d) the expected payoff under the risk-neutral probability measure, discounted at the risk-free rate

4) (5pts) Which strategy is riskiest?
   a) Short butterfly   b) Bear put spread   c) Short call   d) Short put
5) Khary P places the option trades given below, where each option is on Apple and has time-to-expiry 1 month. The current price of Apple stock is $266 (these are real market prices as of 5/3/2010).

- Buy one call with strike 270 and premium 5.65
- Buy two puts with strike 270 and premium 9.00

\[-5.65 - 2(9) = -23.65\]

5a) Carefully draw the profit diagram for this set of trades, showing all strike prices. Use dashed lines for the individual positions (called legs), and a solid line for the net position. (8pts)

5b) What is the maximum profit for this set of trades? (2pts) \[\text{Infinity}\]

Under what condition(s) does this occur? (2pts) \[S_T \to \infty\]

5c) What is the maximum loss for this set of trades? (2pts) \[23.65 \times 100 \text{ shares} = \$2365\]

Under what condition(s) does this occur? (2pts) \[S_T = 270\]

5d) Find the breakeven prices for this strategy. In other words, at expiry, if the stock price falls to \(x\), or if the stock price rises to \(y\), the profit will be exactly zero. Find \(x\) and \(y\), carefully showing all work. (4pts)

\[\begin{align*}
& S_T \leq 270 & \text{Profit} &= \max(S_T - 270, 0) + 2\max(270 - S_T, 0) - 23.65 \\
& & = 2(270 - S_T) - 23.65 & \text{set } 0 \Rightarrow S_T = 258.175 \\
& S_T > 270 & \text{Profit} &= \max(S_T - 270, 0) + 2\max(270 - S_T, 0) - 23.65 \\
& & = S_T - 270 - 23.65 & \text{set } 0 \Rightarrow S_T = 293.65
\end{align*}\]

Thus, at expiry, if the stock price falls to 258.175, or if the stock price rises to 293.65, the profit will be exactly zero.
6. An airline expects to purchase 3 million gallons of jet fuel in one month and decides to use heating oil futures for hedging. Each heating oil contract traded on the NYMEX is on 42,000 gallons of heating oil. Based on the recent price changes in the table below, determine the optimal hedge. State if the hedge position is long or short, and write an integral number of contracts, rounding up (e.g., if we need 66.5 contracts, write 67). (20pts)

\[ h^* = \rho \frac{\sigma_S}{\sigma_F} \]
\[ N^* = \frac{h^* QA}{Q_F} \]

\[ \frac{3}{ \sum_{i=1}^{n} x_i } = 0.61 \]
\[ \frac{3}{ \sum_{i=1}^{n} y_i } = 0.01 \]
\[ \frac{3}{ \sum_{i=1}^{n} x_i^2 } = 0.0045 \]
\[ \frac{3}{ \sum_{i=1}^{n} y_i^2 } = 0.0029 \]
\[ \frac{3}{ \sum_{i=1}^{n} x_i y_i } = 0.0034 \]

\[ n = 3 \]

<table>
<thead>
<tr>
<th>Period</th>
<th>( \Delta ) Change in futures price</th>
<th>( \Delta ) Change in jet fuel price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>-0.05</td>
<td>-0.04</td>
</tr>
<tr>
<td>4</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

\[ \sigma_F = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - (\bar{x} \cdot \bar{x})^2}{n(n-1)}} = 0.047258 \]

\[ \sigma_S = \sqrt{\frac{\sum_{i=1}^{n} y_i^2 - (\bar{y} \cdot \bar{y})^2}{n(n-1)}} = 0.037859 \]

\[ \rho = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{\sqrt{[n \sum_{i=1}^{n} x_i^2 - (\bar{x} \cdot \bar{x})^2]} \sqrt{[n \sum_{i=1}^{n} y_i^2 - (\bar{y} \cdot \bar{y})^2]}} = 0.94085 \]

\[ h^* = \rho \frac{\sigma_S}{\sigma_F} = 0.94085 \frac{0.037859}{0.047258} = 0.753726 \]

\[ Q_A = 3 \text{mil} \quad Q_F = 42,000 \]

\[ \frac{h^* QA}{Q_F} = \frac{0.753726 \times (3 \text{mil})}{42,000} = 53.84 \approx 54. \]

The airline should long 54 future contracts.
7) Consider an American put option with strike price $105$ and time-to-expiry 2 months. The current underlying stock price is $90$. The volatility of the stock is $30\%$. Use a binomial tree with a step size of one month to price this option. Carefully draw and label the tree, naming the nodes, writing stock prices above each node, and option prices below each node. (20pts)

\[ u = e^{\sigma \sqrt{t}} = 1.07 \]  \[ d = \frac{1}{u} = 0.91 \]  \[ k = 105 \]

\[ f_{uu} = 0 \]  \[ f_{ud} = 15.0423 \]  \[ f_{dd} = 28.32 \]

\( @B, \) pay off from early exercise = 6.9

\[ f_{u} = e^{-rT} [ p_{uu} f_{uu} + (1-p) f_{ud} ] = 7.6676 \]  \( \Rightarrow f_{u} = 7.6676 \)  No EE

\( @C, \) pay off from EE = 22.47

\[ f_{d} = e^{-rT} [ p_{dd} f_{dd} + (1-p) f_{ud} ] = 21.785 \]  \( \Rightarrow f_{d} = 22.47 \)  EE is optimal

\( @A, \) pay off from EE = 15

\[ f = e^{-rT} [ p_{d} f_{u} + (1-p) f_{d} ] = 15.2 \]  \( \Rightarrow f^{*} = 15.2 \)  No EE

8a) Stock APT is currently trading at $100$; the stock does not pay dividends. The forward contract to purchase one share of APT stock in six months is $102$. Is there an arbitrage opportunity? If so, clearly write the steps of the arbitrage trade. (10pts)

\[ r = 2\% \]  \[ T = \frac{6}{12} \]  \[ F = 102 \]  \[ S_0 = 100 \]

\[ S_0 e^{rT} = 100 \cdot e^{0.02 \left( \frac{1}{2} \right)} = 101.005 < 102 \]

Yes, there is an arbitrage opportunity.

\begin{enumerate}
  \item Borrow $100 @ r = 2\%$, now.
  \item Buy 1 share of APT stock @$100$, Short 1 forward contract @$102$.
\end{enumerate}

6 months later, exercise the forward contract: gain $102$

\[ \text{pay back money borrowed along with interest: lose} \]

\[ 100 \cdot e^{0.02 \left( \frac{1}{2} \right)} = 101.005 \]

\[ \Rightarrow \text{Risk-free profit} = 102 - 101.005 = 0.995 \]

8b) Consider a European call option and a European put option on APT. Each option has a strike price of 105 and expires in 1 month. If the price of the call option is $1$, find the price of the put option, assuming no arbitrage. (10pts)

\[ p + S_0 = c + k e^{-rT} \]

\[ \Rightarrow P = 1 + 105 e^{-0.02(\frac{1}{12})} - 100 \]

\[ = 5.825 \]

Tell me a joke (keep it clean)