Chapter 15
The Greek Letters

SOLUTIONS TO QUESTIONS AND PROBLEMS

Problem 15.1.
Suppose the strike price is 10.00. The option writer aims to be fully covered whenever the option is in the money and naked whenever it is out of the money. The option writer attempts to achieve this by buying the assets underlying the option as soon as the asset price reaches 10.00 from below and selling as soon as the asset price reaches 10.00 from above. The trouble with this scheme is that it assumes that when the asset price moves from 9.99 to 10.00, the next move will be to a price above 10.00. (In practice the next move might back to 9.99.) Similarly it assumes that when the asset price moves from 10.01 to 10.00, the next move will be to a price below 10.00. (In practice the next move might be back to 10.01.) The scheme can be implemented by buying at 10.01 and selling at 9.99. However, it is not a good hedge. The cost of the trading strategy is zero if the asset price never reaches 10.00 and can be quite high if it reaches 10.00 many times. A good hedge has the property that its cost is always very close the value of the option.

Problem 15.2.
A delta of 0.7 means that, when the price of the stock increases by a small amount, the price of the option increases by 70% of this amount. Similarly, when the price of the stock decreases by a small amount, the price of the option decreases by 70% of this amount. A short position in 1,000 options has a delta of \(-700\) and can be made delta neutral with the purchase of 700 shares.

Problem 15.3.
In this case \(S_0 = K, r = 0.1, \sigma = 0.25, \text{ and } T = 0.5\). Also,

\[
d_1 = \frac{\ln(S_0/K) + (0.1 + 0.25^2/2)0.5}{0.25\sqrt{0.5}} = 0.3712
\]

The delta of the option is \(N(d_1)\) or 0.64.

Problem 15.4.
A theta of \(-0.1\) means that if \(\Delta t\) years pass with no change in either the stock price or its volatility, the value of the option declines by 0.1\(\Delta t\). If a trader feels that neither the stock price
nor its implied volatility will change, he or she should write an option with as high a theta as possible. Relatively short-life at-the-money options have the highest theta.

**Problem 15.5.**

The gamma of an option position is the rate of change of the delta of the position with respect to the asset price. For example, a gamma of 0.1 would indicate that when the asset price increases by a certain small amount delta increases by 0.1 of this amount. When the gamma of an option writer's position is large and negative and the delta is zero, the option writer will lose significant amounts of money if there is a large movement (either an increase or a decrease) in the asset price.

**Problem 15.6.**

To hedge an option position it is necessary to create the opposite option position synthetically. For example, to hedge a long position in a put it is necessary to create a short position in a put synthetically. It follows that the procedure for creating an option position synthetically is the reverse of the procedure for hedging the option position.

**Problem 15.7.**

Portfolio insurance involves creating a put option synthetically. It assumes that as soon as a portfolio's value declines by a small amount the portfolio manager's position is rebalanced by either (a) selling part of the portfolio, or (b) selling index futures. On October 19, 1987, the market declined so quickly that the sort of rebalancing anticipated in portfolio insurance schemes could not be accomplished.

**Problem 15.8.**

The strategy costs the trader $0.20 each time the stock is bought and sold. The total expected cost of the strategy, in present value terms, must be $4. This means that the expected number of times the stock will be bought and sold is approximately 20. The expected number of times it will be bought is approximately 20 and the expected number of times it will be sold is also approximately 20. The buy and sell transactions can take place at any time during the life of the option. The above numbers are therefore only approximately correct because of the effects of discounting. Also they assume a risk-neutral world.

**Problem 15.9.**

The holding of the stock at any given time must be \( N(d_1) \). Hence the stock is bought just after the price has risen and sold just after the price has fallen. (This is the buy high sell low strategy referred to in the text.) In the first scenario the stock is continually bought. In second scenario the stock is bought, sold, bought again, sold again, etc. The final holding is the same in both scenarios. The buy, sell, buy, sell... situation clearly leads to higher costs than the buy, buy, buy... situation. This problem emphasizes one disadvantage of creating options synthetically. Whereas the cost of an option that is purchased is known up front and depends
Delta
\[-1000 \times 0.5 - 500 \times 0.8 - 2000 \times (-0.4) - 500 \times 0.7 = -450\]

Gamma
\[-1000 \times 2.2 - 500 \times 0.6 - 2000 \times 1.3 - 500 \times 1.8 = -6000\]

Vega
\[-1000 \times 1.8 - 500 \times 0.2 - 2000 \times 0.7 - 500 \times 1.4 = -4000\]

a) gamma-neutral: \( w_i \Gamma_i = 6000 \Rightarrow w_i = 4000 \)
derta and gamma neutral: \(-450 + 0.6 \times 4000 = 1950\)
Thus, short position in 1950 sterling and long position in 4000 options would make the portfolio both delta and gamma neutral.

b) vega-neutral: \( \frac{-V}{V_i} = \frac{4000}{0.8} = 5000 \)
delta and vega neutral: \(-450 + 0.6 \times 5000 = 2550\)
Thus, long position in 2550 sterling and long position in 5000 options would make the portfolio both delta and vega neutral.
Chapter 16
Volatility Smiles

SOLUTIONS TO QUESTIONS AND PROBLEMS

Problem 16.1.
When both tails of the stock price distribution are less heavy than those of the lognormal distribution, Black–Scholes will tend to produce relatively high prices for options when they are either significantly out of the money or significantly in the money. This leads to an implied volatility pattern similar to that in Figure 16.7. When the right tail is heavier and the left tail is less heavy, Black–Scholes will tend to produce relatively low prices for out-of-the-money calls and in-the-money puts. It will tend to produce relatively high prices for out-of-the-money puts and in-the-money calls. This leads to implied volatility being an increasing function of strike price.

Problem 16.2.
A downward sloping volatility smile is usually observed for equities.

Problem 16.3.
Jumps tend to make both tails of the stock price distribution heavier than those of the lognormal distribution. This creates a volatility smile similar to that in Figure 16.1. The volatility smile is likely to be more pronounced for the three-month option.

Problem 16.4.
The put is has a price that is too low relative to the call's price. The correct trading strategy is to buy the put, buy the stock and sell the call.

Problem 16.5.
The heavier left tail should lead to high prices, and therefore high implied volatilities, for out-of-the-money (low-strike-price) puts. Similarly the less heavy right tail should lead to low prices, and therefore low volatilities for out-of-the-money (high-strike-price) calls. A volatility smile where volatility is a decreasing function of strike price results.

Problem 16.6.
With the notation in the text

\[ c_{bs} + Ke^{-rT} = p_{bs} + Se^{-qT} \]
\[ c_{\text{mkt}} + Ke^{-rT} = p_{\text{mkt}} + Se^{-qT} \]

It follows that
\[ c_{\text{bs}} - c_{\text{mkt}} = p_{\text{bs}} - p_{\text{mkt}} \]

In this case \( c_{\text{mkt}} = 3.00; c_{\text{bs}} = 3.50; \) and \( p_{\text{bs}} = 1.00. \) It follows that \( p_{\text{mkt}} \) should be 0.50.

**Problem 16.7.**

The crashophobia argument is an attempt to explain the pronounced volatility skew in equity markets since 1987. (This was the year equity markets shocked everyone by crashing more than 20% in one day.) The argument is that traders are concerned about another crash and as a result increase the price of out-of-the-money puts. This creates the volatility skew.

**Problem 16.8.**

The probability distribution of the stock price in one month is not lognormal. Possibly it consists of two lognormal distributions superimposed upon each other and is bimodal. Black–Scholes is clearly inappropriate, because it assumes that the stock price at any future time is lognormal.

**Problem 16.9.**

When the asset price is positively correlated with volatility, the volatility tends to increase as the asset price increases, producing thin left tails and fat right tails. Implied volatility then increases with the strike price.

**Problem 16.10.**

There are a number of problems in testing an option pricing model empirically. These include the problem of obtaining synchronous data on stock prices and option prices, the problem of estimating the dividends that will be paid on the stock during the option’s life, the problem of distinguishing between situations where the market is inefficient and situations where the option pricing model is incorrect, and the problems of estimating stock price volatility.

**Problem 16.11.**

In this case the probability distribution of the exchange rate has a thin left tail and a thin right tail relative to the lognormal distribution. We are in the opposite situation to that described for foreign currencies in Section 16.2. Both out-of-the-money and in-the-money calls and puts can be expected to have lower implied volatilities than at-the-money calls and puts. The pattern of implied volatilities is likely to be similar to Figure 16.7.

**Problem 16.12.**

A deep-out-of-the-money option has a low value. Decreases in its volatility reduce its value. However, this reduction is small because the value can never go below zero. Increases in its volatility, on the other hand, can lead to significant percentage increases in the value of the option. The option does, therefore, have some of the same attributes as an option on volatility.