Problem 3.17.
Suppose that the weather is bad and the farmer’s production is lower than expected. Other farmers are likely to have been affected similarly. Corn production overall will be low and as a consequence the price of corn will be relatively high. The farmer is likely to be overhedged relative to actual production. The farmer’s problems arising from the bad harvest will be made worse by losses on the short futures position. This problem emphasizes the importance of looking at the big picture when hedging. The farmer is correct to question whether hedging price risk while ignoring other risks is a good strategy.

Problem 3.18.
A short position in
\[
1.3 \times \frac{50,000 \times 30}{50 \times 1,500} = 26
\]
contracts is required.

Problem 3.19.
If the company uses a hedge ratio of 1.5 in Table 3.5 it would at each stage short 150 contracts. The gain from the futures contracts would be
\[
1.50 \times 1.70 = $2.55 \text{ per barrel}
\]
and the company would be $0.85 per barrel better off.

Problem 3.20.
Suppose that you enter into a short futures contract to hedge the sale of an asset in six months. If the price of the asset rises sharply during the six months, the futures price will also rise and you may get margin calls. The margin calls will lead to cash outflows. Eventually the cash outflows will be offset by the extra amount you get when you sell the asset, but there is a mismatch in the timing of the cash outflows and inflows. Your cash outflows occur earlier than your cash inflows. A similar situation could arise if you used a long position in a futures contract to hedge the purchase of an asset and the asset’s price fell sharply. An extreme example of what we are talking about here is provided by Metallgesellschaft (see Business Snapshot 3.2).

Problem 3.21.
It may well be true that there is just as much chance that the price of oil in the future will be above the futures price as that it will be below the futures price. This means that the use of a futures contract for speculation would be like betting on whether a coin comes up heads or tails. But it might make sense for the airline to use futures for hedging rather than speculation. The futures contract then has the effect of reducing risks. It can be argued that an airline should not expose its shareholders to risks associated with the future price of oil when there are contracts available to hedge the risks.
3.25

a)

\[ P = 50\text{mil} \quad F = 250 \times 1250 = 312,500 \quad \beta = 0.87 \]

\[ N^* = \beta \frac{P}{F} = 0.87 \times \frac{50\text{mil}}{312,500} = 139.2 \]

Therefore, the fund manager should short 139 future contracts to hedge the portfolio.
Chapter 5
Determination of Forward and Futures Prices

SOLUTIONS TO QUESTIONS AND PROBLEMS

Problem 5.1.
The investor's broker borrows the shares from another client's account and sells them in the usual way. To close out the position, the investor must purchase the shares. The broker then replaces them in the account of the client from whom they were borrowed. The party with the short position must remit to the broker dividends and other income paid on the shares. The broker transfers these funds to the account of the client from whom the shares were borrowed. Occasionally the broker runs out of places from which to borrow the shares. The investor is then short squeezed and has to close out the position immediately.

Problem 5.2.
The forward price of an asset today is the price at which you would agree to buy or sell the asset at a future time. The value of a forward contract is zero when you first enter into it. As time passes the underlying asset price changes and the value of the contract may become positive or negative.

Problem 5.3.
The forward price is

$$30e^{0.12 \times 0.5} = \$31.86$$

Problem 5.4.
The futures price is

$$350e^{(0.08 - 0.04) \times 0.3333} = \$354.7$$

Problem 5.5.
Gold is an investment asset. If the futures price is too high, investors will find it profitable to increase their holdings of gold and short futures contracts. If the futures price is too low, they will find it profitable to decrease their holdings of gold and go long in the futures market. Copper is a consumption asset. If the futures price is too high, a strategy of buy copper and short futures works. However, because investors do not in general hold the asset, the strategy
Problem 5.10.

Using equation (5.3) the six month futures price is

\[ 150e^{(0.07 - 0.032) \times 0.5} = 152.88 \]

or $152.88.

Problem 5.11.

The futures contract lasts for five months. The dividend yield is 2% for three of the months and 5% for two of the months. The average dividend yield is therefore

\[ \frac{1}{5} (3 \times 2 + 2 \times 5) = 3.2\% \]

The futures price is therefore

\[ 300e^{(0.09 - 0.032) \times 0.4167} = 307.34 \]

or $307.34.

Problem 5.12.

The theoretical futures price is

\[ 400e^{(0.10 - 0.04) \times 4/12} = 408.08 \]

The actual futures price is only 405. This shows that the index futures price is too low relative to the index. The correct arbitrage strategy is

1. Buy futures contracts
2. Short the shares underlying the index.

Problem 5.13.

The settlement prices for the futures contracts are

Mar 0.08920
June 0.08812

The June 2004 price is about 1.2% below the March 2004 price. This suggests that the short-term interest rate in the Mexico exceeded short-term interest rates in the United States by about 1.2% per three months or about 4.8% per year.

Problem 5.14.

The theoretical futures price is

\[ 0.6500e^{(0.08 - 0.03) \times 2/12} = 0.6554 \]
The actual futures price is too high. This suggests that an arbitrageur should buy Swiss francs and short Swiss francs futures.

**Problem 5.15.**

The present value of the storage costs for nine months are

\[ 0.06 + 0.06e^{-0.10 \times 0.25} + 0.06e^{-0.10 \times 0.5} = 0.176 \]

or $0.176. The futures price is from equation (5.11) given by \( F_0 \) where

\[ F_0 = (9.000 + 0.176)e^{0.1 \times 0.75} = 9.89 \]

i.e., it is $9.89 per ounce.

**Problem 5.16.**

If

\[ F_2 > F_1 e^{r(t_2-t_1)} \]

an investor could make a riskless profit by

1. Taking a long position in a futures contract which matures at time \( t_1 \)
2. Taking a short position in a futures contract which matures at time \( t_2 \)

When the first futures contract matures, the asset is purchased for \( F_1 \) using funds borrowed at rate \( r \). It is then held until time \( t_2 \) at which point it is exchanged for \( F_2 \) under the second contract. The costs of the funds borrowed and accumulated interest at time \( t_2 \) is \( F_1 e^{r(t_2-t_1)} \) A positive profit of

\[ F_2 - F_1 e^{r(t_2-t_1)} \]

is then realized at time \( t_2 \). This type of arbitrage opportunity cannot exist for long. Hence:

\[ F_2 \leq F_1 e^{r(t_2-t_1)} \]

**Problem 5.17.**

In total the gain or loss under a futures contract is equal to the gain or loss under the corresponding forward contract. However the timing of the cash flows is different. When the time value of money is taken into account a futures contract may prove to be more valuable or less valuable than a forward contract. Of course the company does not know in advance which will work out better. The long forward contract provides a perfect hedge. The long futures contract provides a slightly imperfect hedge.

(a) In this case the forward contract would lead to a slightly better outcome. The company will make a loss on its hedge. If the hedge is with a forward contract the whole of the loss will be realized at the end. If it is with a futures contract the loss will be realized day by day throughout the contract. On a present value basis the former is preferable.
Problem 10.5.

A reverse calendar spread is created by buying a short-maturity option and selling a long-maturity option, both with the same strike price.

Problem 10.6.

Both a straddle and a strangle are created by combining a long position in a call with a long position in a put. In a straddle the two have the same strike price and expiration date. In a strangle they have different strike prices and the same expiration date.

Problem 10.7.

A strangle is created by buying both options. The pattern of profits is as follows:

<table>
<thead>
<tr>
<th>Stock Price $S_T$</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T &lt; 45$</td>
<td>$(45 - S_T) - 5$</td>
</tr>
<tr>
<td>$45 &lt; S_T &lt; 50$</td>
<td>$-5$</td>
</tr>
<tr>
<td>$S_T &gt; 50$</td>
<td>$(S_T - 50) - 5$</td>
</tr>
</tbody>
</table>

Problem 10.8.

A bull spread using calls provides a profit pattern with the same general shape as a bull spread using puts (see Figures 10.2 and 10.3 in the text). Define $p_1$ and $c_1$ as the prices of put and call with strike price $K_1$ and $p_2$ and $c_2$ as the prices of a put and call with strike price $K_2$. From put-call parity

\[ p_1 + S = c_1 + K_1 e^{-rT} \]
\[ p_2 + S = c_2 + K_2 e^{-rT} \]

Hence:

\[ p_1 - p_2 = c_1 - c_2 - (K_2 - K_1) e^{-rT} \]

This shows that the initial investment when the spread is created from puts is less than the initial investment when it is created from calls by an amount $(K_2 - K_1) e^{-rT}$. In fact as mentioned in the text the initial investment when the bull spread is created from puts is negative, while the initial investment when it is created from calls is positive.

The profit when calls are used to create the bull spread is higher than when puts are used by $(K_2 - K_1) (1 - e^{-rT})$. This reflects the fact that the call strategy involves an additional risk-free investment of $(K_2 - K_1) e^{-rT}$ over the put strategy. This earns interest of $(K_2 - K_1) e^{-rT} (e^{rT} - 1) = (K_2 - K_1)(1 - e^{-rT})$.

Problem 10.9.

An aggressive bull spread using call options is discussed in the text. Both of the options used have relatively high strike prices. Similarly, an aggressive bear spread can be created using put options. Both of the options should be out of the money (that is, they should have relatively
low strike prices). The spread then costs very little to set up because both of the puts are worth close to zero. In most circumstances the spread will provide zero payoff. However, there is a small chance that the stock price will fall fast so that on expiration both options will be in the money. The spread then provides a payoff equal to the difference between the two strike prices, \( K_2 - K_1 \).

**Problem 10.10.**

A bull spread is created by buying the $30 put and selling the $35 put. This strategy gives rise to an initial cash inflow of $3. The outcome is as follows:

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>Payoff</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_T \geq 35 )</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>( 30 \leq S_T &lt; 35 )</td>
<td>( S_T - 35 )</td>
<td>( S_T - 32 )</td>
</tr>
<tr>
<td>( S_T &lt; 30 )</td>
<td>(-5)</td>
<td>(-2)</td>
</tr>
</tbody>
</table>

A bear spread is created by selling the $30 put and buying the $35 put. This strategy costs $3 initially. The outcome is as follows:

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>Payoff</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_T \geq 35 )</td>
<td>0</td>
<td>(-3)</td>
</tr>
<tr>
<td>( 30 \leq S_T &lt; 35 )</td>
<td>( 35 - S_T )</td>
<td>( 32 - S_T )</td>
</tr>
<tr>
<td>( S_T &lt; 30 )</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

**Problem 10.11.**

Define \( c_1 \), \( c_2 \), and \( c_3 \) as the prices of calls with strike prices \( K_1 \), \( K_2 \) and \( K_3 \). Define \( p_1 \), \( p_2 \) and \( p_3 \) as the prices of puts with strike prices \( K_1 \), \( K_2 \) and \( K_3 \). With the usual notation

\[
\begin{align*}
  c_1 + K_1 e^{-rT} &= p_1 + S \\
  c_2 + K_2 e^{-rT} &= p_2 + S \\
  c_3 + K_3 e^{-rT} &= p_3 + S 
\end{align*}
\]

Hence

\[
\begin{align*}
  c_1 + c_3 - 2c_2 + (K_1 + K_3 - 2K_2)e^{-rT} &= p_1 + p_3 - 2p_2 
\end{align*}
\]

Because \( K_2 - K_1 = K_3 - K_2 \), it follows that \( K_1 + K_3 - 2K_2 = 0 \) and

\[
\begin{align*}
  c_1 + c_3 - 2c_2 &= p_1 + p_3 - 2p_2 
\end{align*}
\]

The cost of a butterfly spread created using European calls is therefore exactly the same as the cost of a butterfly spread created using European puts.

**Problem 10.12.**

A straddle is created by buying both the call and the put. This strategy costs $10. The profit/loss is shown in the following table:
<table>
<thead>
<tr>
<th>Stock Price</th>
<th>Payoff</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T &gt; 60$</td>
<td>$S_T - 60$</td>
<td>$S_T - 70$</td>
</tr>
<tr>
<td>$S_T \leq 60$</td>
<td>$60 - S_T$</td>
<td>$50 - S_T$</td>
</tr>
</tbody>
</table>

This shows that the straddle will lead to a loss if the final stock price is between $50$ and $70$.

**Problem 10.13.**

The bull spread is created by buying a put with strike price $K_1$ and selling a put with strike price $K_2$. The payoff is calculated as follows:

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>Payoff from Long Put Option</th>
<th>Payoff from Short Put Option</th>
<th>Total Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T \geq K_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$K_1 &lt; S_T &lt; K_2$</td>
<td>0</td>
<td>$S_T - K_2$</td>
<td>$-(K_2 - S_T)$</td>
</tr>
<tr>
<td>$S_T \leq K_1$</td>
<td>$K_1 - S_T$</td>
<td>$S_T - K_2$</td>
<td>$-(K_2 - K_1)$</td>
</tr>
</tbody>
</table>

**Problem 10.14.**

Possible strategies are:

- Strangle
- Straddle
- Strip
- Strap
- Reverse calendar spread
- Reverse butterfly spread

The strategies all provide positive profits when there are large stock price moves. A strangle is less expensive than a straddle, but requires a bigger move in the stock price in order to provide a positive profit. Strips and straps are more expensive than straddles but provide bigger profits in certain circumstances. A strip will provide a bigger profit when there is a large downward stock price move. A strap will provide a bigger profit when there is a large upward stock price move. In the case of strangles, straddles, strips and straps, the profit increases as the size of the stock price movement increases. By contrast in a reverse calendar spread and a reverse butterfly spread there is a maximum potential profit regardless of the size of the stock price movement.

**Problem 10.15.**

Suppose that the delivery price is $K$ and the delivery date is $T$. The forward contract is created by buying a European call and selling a European put when both options have strike price $K$ and exercise date $T$. This portfolio provides a payoff of $S_T - K$ under all circumstances where $S_T$ is the stock price at time $T$. Suppose that $F_0$ is the forward price. If $K = F_0$, the forward contract that is created has zero value. This shows that the price of a call equals the price of a put when the strike price is $F_0$. 
A butterfly spread can be created by buying a call option with a $55 strike price, buying one call with a $65 strike price, and selling two calls with a $60 strike price.

It can also be created using put options. The investor buys one put option with a $55 strike price, another with a $65 strike price, and sells two puts with a $60 strike price.

<table>
<thead>
<tr>
<th>Stock price Range</th>
<th>Payoff from 1st long call</th>
<th>Payoff from 2nd long call</th>
<th>Payoff from Short calls</th>
<th>Total payoff</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T \leq 55</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>55 &lt; $S_T &lt; 60</td>
<td>$S_T - 55</td>
<td>0</td>
<td>0</td>
<td>$S_T - 55</td>
<td>$S_T - 55 - 1</td>
</tr>
<tr>
<td>60 &lt; $S_T &lt; 65</td>
<td>$S_T - 55</td>
<td>0</td>
<td>-2 ($S_T - 60)</td>
<td>65 - $S_T</td>
<td>65 - $S_T - 1</td>
</tr>
<tr>
<td>$S_T \geq 65</td>
<td>$S_T - 55</td>
<td>$S_T - 65</td>
<td>-2 ($S_T - 60)</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

If the stock price in the future is greater than $65 or less than $55, the total payoff is zero, and the investor incurs a net loss of $1. If the stock price is between $56 and $64, a profit is made. The maximum profit, $4, occurs when the stock price in the future is $60.