A regime-switching approach to model-based stress testing

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The current financial marketplace has seen the development of increasingly complex products and proper risk management has become a challenging task. Accurate stress testing relies upon models that capture their essential features. Novel, powerful ideas have emerged regarding the dynamics of financial instruments, such as incorporating mechanisms to model regime-switching and non-linearity. This paper outlines the logistic mixture of the linear components (LMLC) model and illustrates how it can be used to perform stress testing on an asset that follows regime-switching dynamics. Once the model is estimated, running a stress test is straightforward; expected loss estimates are produced from the fitting equation. A market stress test is run on a merger arbitrage hedge fund position. It is demonstrated that the LMLC stress test produces expected loss estimates that are more conservative than estimates produced from a linear factor model.

1 INTRODUCTION

Stress testing is a routine risk-management tool used in conjunction with objective models such as value-at-risk (VaR). The practice involves specifying extreme conditions for underlying risks (e.g., market, credit) and measuring the effect that it has on capital. A large body of literature exists on this topic; an overview can be found in Drehmann (2008).

Stress tests have received considerable attention lately with the US government’s stress testing of banks with more than US$100 billion in assets. This process, which began February 25, 2009, examined how banks would fare given stressful settings of core economic variables, such as a fall in housing prices of 22% and an unemployment rate of 8.9% over the current year. The Basel Committee on Banking Supervision (2005) requires banks to perform stress tests that are plausible, severe and suggestive for the design of risk-reducing action. In January 2009, the committee issued a paper outlining principles for the sound design and implementation of stress testing programs at banks.

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One of the challenges in developing an accurate stress test is to construct a model that preserves the essential features of the data while keeping it parsimonious. At present, there are an array of financial assets with a high degree of complexity (e.g., derivatives, structured products). Relating the rate of return of these assets to underlying stress factors is not a straightforward task and quite often linear factor models (e.g., the capital asset pricing model) fail to adequately explain the relationship. A large body of research exists that documents the non-linear relationship of hedge fund returns to underlying factors and proposes alternative models. This includes Fung and Hsieh (1997, 2001), Mitchell and Pulvino (2001), Agarwal and Naik (2004) and Tashman and Frey (2009). Drehmann (2008) addresses the issue that non-linearities may emerge during stress and that one possibility could be to model them using regime switching:

Non-linearities may also arise because of jumps or switching between multiple equilibria. Another possibility could be to capture them by models with regime shifts. It seems a fruitful avenue for future research to assess how important regime shifts are and whether they could be practically incorporated into risk management and stress testing models.

Given a financial time series comprised of quiet and hectic data points, it may be tempting to filter the data and fit a model to the hectic points only. In this way, the analyst could presumably better predict the response variable in a period of economic stress, while circumventing the use of a more complex model (e.g., a regime-switching model). Boyer et al. (1999) demonstrate that this technique can yield misleading results as a consequence of selection bias. They argue that “it is necessary for the researcher to begin with a data-coherent model of the data generating process that builds in the possibility of structural changes, and then estimate the model’s parameters.” The main contribution of this paper is that it proposes such a data-coherent model for stress testing.

1.1 Mixture models in finance

A mixture model is a probability density function that is a convex combination of other probability density functions. It is commonly used when parametric approaches fail to capture important features of the data. For example, a random variable with a bimodal distribution may be modeled by a mixture of two normals. See McLachlan and Peel (2000) for a comprehensive review of mixture modeling.

Mixture modeling has been applied broadly in finance to problems such as asset pricing and risk management. There have been numerous studies of hedge fund returns using mixture models, as their return distributions are generally non-normal. Regime-switching has been used to provide dynamic models for assets following a mixture distribution (Hamilton (1989), Lopez de Prado and Peijan (2004), Bruche and Gonzalez-Aguado (2008), Tashman and Frey (2009)).

Regime-switching models have been applied in financial risk management. Bruche and Gonzalez-Aguado (2008) proposed an econometric model in which the
joint distribution of default and recovery rates was driven by an unobserved Markov
chain. This chain was interpreted as the credit cycle. The approach was shown to
outperform models with only observed macroeconomic variables. Kim and Finger
(2000) proposed the broken arrow technique to improve hypothetical stress tests.
They stipulate that the asset return distribution for a given day depends on whether
the day is quiet or hectic. This setup leads to a mixture-model formulation.

The present paper demonstrates how the logistic mixture of linear components
(LMLC) model can be used as a tool for stress testing. The model first appeared in
Tashman and Frey (2009), where it was shown how the model could be fit and applied
to reduce risk in a hedge fund position. The LMLC stress test will be demonstrated on
a hedge fund position. A hedge fund of funds must run stress tests on its hedge fund
positions as part of its standard risk management practices. Among other things, this
involves applying shocks to market and credit factors, and measuring the expected
loss of the hedge fund positions in its portfolios. These expected losses may then
be aggregated across portfolios to measure the firmwide sensitivity of capital to the
stress scenario. These scenarios include the use of historical settings for the factors
during periods of stress as well as hypothetical settings.

The remainder of the paper proceeds as follows: Section 2 describes the structure
of the LMLC model. It discusses a test to determine when the model is appropriate
and it explains how it can be used to produce estimates for expected loss. Section 3
discusses a stress test applied to a hedge fund position. Section 4 provides results
from this test. Section 5 concludes.

2 OVERVIEW OF THE LMLC MODEL

As mentioned earlier, the LMLC model uses a regime-switching approach. Let
n denote the number of observations. Each observation of the response variable
y = (y_1, y_2, ..., y_n)^T is generated from a regime (in this context, y represents
hedge fund returns). It is not known in advance which regime will generate the
observation. Rather, at the present time, a set of stress factors will determine
the probability that each regime is responsible for producing the observation (the
prior probabilities). This structure is referred to as the mixing component. The
model that produces these prior probabilities is the logistic function. Let v_j be a
d-dimensional vector of measurements taken on a set of stress factors at time j and
let v = (v_1^T, v_2^T, ..., v_n^T)^T.

The prior probabilities at a given time j are the following:

\[ \pi_j(v_j | \alpha) = \left[ \frac{e^{\alpha^T v_j}}{1 + e^{\alpha^T v_j}}, \frac{1}{1 + e^{\alpha^T v_j}} \right], \quad j = 1, \ldots, n \]

where \( \alpha \) is a column vector of parameters. These parameters capture the sensitivity
of the prior probabilities to the stress factors \( v \).

Throughout this analysis, an LMLC model with two regimes is used; one regime
captures data from a quiet period and the other captures data from a hectic period.
Therefore, at each point in time \( j \), there are two prior probabilities \( \pi_{1j} \) and \( \pi_{2j} = 1 - \pi_{1j} \) collected into the vector \( \pi_j \). In each regime, a linear regression function will relate a set of stress factors to the expected response. The covariance structure will be different in each regime, providing a more flexible alternative to standard regression techniques. The regression functions will be referred to as linear components. In its full generality, the stress factors appearing in the mixing component \( v \) can be distinct from the stress factors appearing in each of the linear components. Let the number of stress factors in linear component 1 be \( q \) and the number of stress factors in linear component 2 be \( r \), and potentially \( d \neq q \neq r \). The vector of measurements taken on the stress factors in linear component 1 at time \( j \) is written \( x^{(1)}_j \) and that of measurements on the stress factors in linear component 2 is written \( x^{(2)}_j \). Let \( x^{(1)} = (x^{(1)T}_1, x^{(1)T}_2, \ldots, x^{(1)T}_n)^T \) and \( x^{(2)} = (x^{(2)T}_1, x^{(2)T}_2, \ldots, x^{(2)T}_n)^T \). That is, \( x^{(1)} \) is an \( n \times q \) matrix and \( x^{(2)} \) is an \( n \times r \) matrix of measurements taken on the stress factors in linear components 1 and 2, respectively.

Given regime \( i \), the linear component asserts that the sensitivities of the expected response \( y \) to the stress factors \( x^{(i)} \) are collected in the vector \( \beta^{(i)} \). Mathematically, \( E_i(y \mid x^{(i)}_j, \beta^{(i)}, \sigma^2_i) \) denotes the expectation in the \( i \)th regime.

Denote the probability of observing the response in the \( i \)th regime conditional on certain stress factor settings at time \( j \) as \( f_i(y_j \mid x^{(i)}_j, \beta^{(i)}, \sigma^2_i) \), where \( \beta^{(i)} \) is the parameter vector of sensitivities and \( \sigma^2_i \) is the error variance. It is assumed that conditional on the regime, the errors are normally distributed. Then, this probability is expressed as:

\[
 f_i(y_j \mid x^{(i)}_j, \beta^{(i)}, \sigma^2_i) = \frac{1}{\sqrt{2\pi \sigma^2_i}} \exp\left\{ \frac{(y_j - \beta^{(i)T} x^{(i)}_j)^2}{2\sigma^2_i} \right\}
\]

Estimating the error variance (this estimate will be referred to as the residual variance) of each linear component is similar to the standard regression case, with one difference. In the estimation procedure, the squared residual of each observation is weighted by the posterior probability that it was generated from the given regime. The formula is the following:

\[
 \hat{\sigma}^2_i = \frac{\sum_{j=1}^n \tau_{ij} (y_j - \beta^{(i)T} x^{(i)}_j)^2}{\sum_{j=1}^n \tau_{ij}}
\]

where the posterior probability that the \( i \)th regime is responsible for the \( j \)th observation is calculated as:

\[
 \tau_{ij} = \frac{\pi_{ij}(v_j \mid \alpha) f_i(y_j \mid x^{(i)}_j, \beta^{(i)}, \sigma^2_i)}{\sum_{i=1}^2 \pi_{ij}(v_j \mid \alpha) f_i(y_j \mid x^{(i)}_j, \beta^{(i)}, \sigma^2_i)}
\]

For further details on fitting the LMLC model, see Tashman and Frey (2009).
2.1 Estimating expected loss

Once the LMLC model is fit, expected loss estimates are produced according to the following procedure:

1) Compute the prior probabilities \( \pi_j(v_j | \alpha) = \{\pi_1j, \pi_2j\} \), where \( \pi_2j = 1 - \pi_1j \).
2) For each regime, compute the expected loss. This is referred to as expected loss conditional on regime. For the \( i \)th regime, this is computed as:

\[
E_i(y_j | x_j^{(i)}) = \beta^{(i)T} x_j^{(i)}
\]

3) The expected loss is the probability-weighted expected loss conditional on regime:

\[
E(y_j | v_j, x_j^{(1)}, x_j^{(2)}) = \pi_1jE_1(y_j | x_j^{(1)}) + \pi_2jE_2(y_j | x_j^{(2)})
\]

\[
= \left( \frac{e^{\alpha^T v_j}}{1 + e^{\alpha^T v_j}} \right) \beta^{(1)T} x_j^{(1)} + \left( \frac{1}{1 + e^{\alpha^T v_j}} \right) \beta^{(2)T} x_j^{(2)}
\]

This is referred to as the (two-regime) LMLC fitting equation. Note that negative values indicate expected losses and positive values indicate expected gains.

2.2 Testing for the presence of multiple regimes

The LMLC model makes the assumption that there are multiple underlying regimes driving the response variable. Therefore, before applying the model, it is imperative to test for statistical evidence of a mixture distribution. The test will be the following:

\( H_0: \) response generated from one linear component with normally distributed errors.

\( H_1: \) response generated from a logistic mixture of two linear components with normally distributed errors.

The Bayes information criterion (denoted as BIC in the equation below) was used for model selection and is defined as:

\[
BIC = -2l + p \log n
\]

where \( l \) is the loglikelihood, \( p \) is the number of parameters in the model and \( \log \) is the natural logarithm. From the definition, the model with the more negative Bayes information criterion is preferable.

The loglikelihood under the null hypothesis is the following:

\[
l_{H_0}(\Psi_{H_0} | x_j, y_j) = -\frac{n}{2} \log(2\pi \sigma^2) - \frac{1}{2} \sum_{j=1}^{n} \frac{(y_j - \beta^T x_j)^2}{\sigma^2}
\]
where $\Psi_{H_0} = \{ \beta, \sigma^2 \}$. To arrive at the loglikelihood under the alternative hypothesis, we need the density function of the response variable given the LMLC model. This density is the following:

$$f(y_j | v_j, x_j^{(1)}, x_j^{(2)}, \sigma_1^2, \sigma_2^2) = \sum_{i=1}^{2} \pi_{ij}(v_j | \alpha) f_i(y_j | x_j^{(i)} \beta^{(i)}, \sigma_i^2)$$

Then the loglikelihood under the alternative hypothesis is the following:

$$l_{H_1}(\Psi_{H_1} | v_j, x_j^{(1)}, x_j^{(2)}, y_j) = \sum_{j=1}^{n} \log \left( \frac{e^{\alpha^T v_j}}{1 + e^{\alpha^T v_j}} \right) + \sum_{j=1}^{n} \log \left( \frac{1}{1 + e^{\alpha^T v_j}} \right)$$

$$- \frac{n}{2} \log(2\pi \sigma_1^2) - \frac{1}{2} \sum_{j=1}^{n} \frac{(y_j - \beta^{(1)^T} x_j^{(1)})^2}{\sigma_1^2}$$

$$- \frac{n}{2} \log(2\pi \sigma_2^2) - \frac{1}{2} \sum_{j=1}^{n} \frac{(y_j - \beta^{(2)^T} x_j^{(2)})^2}{\sigma_2^2}$$

where $\Psi_{H_1} = \{ \alpha, \beta^{(1)}, \beta^{(2)}, \sigma_1^2, \sigma_2^2 \}$.

### 2.3 Providing a benchmark

In the case where data follows a mixture distribution, the observations cannot be regarded as independent and thus linear factor models are not appropriate. Nonetheless, a linear factor model is included in this analysis to provide a simple benchmark for the stress test. Subsequently referred to as the benchmark model, it will contain each of the stress factors included in the LMLC model. It is fit to historical data using standard ordinary least squares techniques. Expected loss estimates are produced by applying stress factor settings in the fitted model.

### 3 A MARKET STRESS TEST

#### 3.1 Sample considered

The LMLC stress test will be demonstrated on the merger arbitrage hedge fund strategy. The stress factor considered will be the stock market and the S&P 500 index will be used as a proxy. This is a one-factor stress test; the proposed technique can be easily extended to a multi-factor stress test (to include credit, for instance), but the one-factor case is considered to illustrate the ideas in a simple manner. Initially, $v = x^{(1)} = x^{(2)}$ and variables are removed by backward selection as necessary. The Hedge Fund Research, Inc. Monthly Indices (HFRI) measure the monthly rate of return (subsequently referred to as the performance) of an equally-weighted pool of constituent hedge funds. The HFRI merger arbitrage index will be used as a performance proxy for the hedge fund strategy.
The merger arbitrage strategy was selected for this example because it has a non-linear relationship to stock market returns that is well documented (Mitchell and Pulvino (2001) and Tashman and Frey (2009)). In particular, merger arbitrage returns are uncorrelated to stock market returns when the latter are flat or positive. However, when stock market returns are negative, the correlation is strong and positive. Thus, there is evidence of two regimes: one quiet (the case of no correlation) and one hectic (the case of strong correlation).

Historical monthly returns of the HFRI merger arbitrage index and the S&P 500 index were used to fit the models (the in-sample data), and the time period considered was January 1990–December 2007, for 216 data points. This period includes several stress events, such as the 1998 Russian financial crisis and September 11, 2001. The starting point of January 1990 represents the inception date of the hedge fund index.

### 3.2 Out-of-sample evaluation

After the models were fit to the sample of historical data through December 2007, a historical stress test was run and evaluated out-of-sample. Specifically, expected loss numbers were calculated for the merger arbitrage index during each month in 2008, given the realized returns of the S&P 500. For months when the S&P 500 recorded a gain, the model-based estimates will likely produce expected gains for the merger arbitrage index; these months are included for completeness. This procedure was done for both the LMLC model and the benchmark model. To date, 2008 was the most difficult year for the merger arbitrage strategy and the worst year for the US stock market since the Great Depression. Thus, the historical stress test evaluates merger arbitrage performance during an extreme period.

Next, the model-based expected losses were compared to the realized merger arbitrage losses in 2008. To quantify some important features of the stress test performance, the following statistics are reported:

- **HitRatio**: this measure represents the percentage of times when the merger arbitrage index realized a loss and the model correctly expected a loss to occur:

  \[
  \text{HitRatio} = \frac{\sum_{j=n+1}^{n+12} I_{(-\infty,0)}(y_j) I_{(-\infty,0)}(E(y_j | v_j, x_j^{(1)}, x_j^{(2)}))}{\sum_{j=n+1}^{n+12} I_{(-\infty,0)}(y_j)}
  \]

  where \( I \) is the indicator function defined as:

  \[
  I_A(x) = \begin{cases} 
  1 & \text{if } x \in A \\
  0 & \text{if } x \notin A
  \end{cases}
  \]

  Note that this statistic measures how accurately the model estimates the direction of the realized return, but not its magnitude.
• LossUndershoot: this measure represents the percentage of times when the model expected a less severe loss than the realized loss:

\[
\text{LossUndershoot} = \frac{\sum_{j=n+1}^{n+12} I_{(-\infty,0)}(y_j) I_{(y_j, +\infty)}(E(y_j | v_j, x_j^{(1)}, x_j^{(2)}))}{\sum_{j=n+1}^{n+12} I_{(-\infty,0)}(y_j)}
\]

This statistic reflects the idea that while it is preferable for the model to accurately estimate direction and magnitude of losses, it is better (from a risk-management perspective) to overestimate the size of a loss than to underestimate it.

4 RESULTS

Figure 1 presents a scatterplot of HFRI merger arbitrage index returns versus S&P 500 index returns over the in-sample period Jan 1990–Dec 2007. There is a clearly non-linear pattern: the merger arbitrage returns are directly related when the S&P 500 returns are negative and uncorrelated when the S&P 500 returns are positive.

In testing for the number of regimes, the Bayes information criterion was $-1,182$ and $-1,294$ for the benchmark model and the LMLC model, respectively. Thus, there is statistical support for the presence of two regimes versus a single regime and we can proceed to apply the LMLC model.

Table 1 (see page 9) shows the LMLC model. All intercepts and slopes in the model are significant at the $\alpha = 0.05$ level. Linear component 1 includes only an
TABLE 1 LMLC model fit to HFRI merger arbitrage index, January 1990–December 2007.

<table>
<thead>
<tr>
<th>Parameter estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixing component</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1.83</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>46.18</td>
</tr>
<tr>
<td>Linear component 1</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>$1.23 \times 10^{-2}$</td>
</tr>
<tr>
<td>Residual variance</td>
<td>$8.53 \times 10^{-3}$</td>
</tr>
<tr>
<td>Linear component 2</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>$-9.24 \times 10^{-3}$</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.29</td>
</tr>
<tr>
<td>Residual variance</td>
<td>$1.91 \times 10^{-2}$</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Parameter estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$7.12 \times 10^{-3}$</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.14</td>
</tr>
</tbody>
</table>

The S&P 500 factor in the mixing component (a slope term) has a parameter estimate with positive sign. This means that there is a direct relationship between stock market returns and the probability that merger arbitrage returns are generated from the first regime. Thus, as stock market returns increase, it becomes more likely that the system is in the first regime, where expected returns are unaffected by the market risk factor.

The residual variance in the second regime is five times as great as the residual variance in the first regime. Thus, in addition to the presence of higher correlation between the stress factor and the hedge fund performance in the second regime, there is greater uncertainty of returns not explained by this factor. Given the structure of
the model, the first regime will be referred to as the quiet regime and the second regime as the hectic regime.

Now that the LMLC model has been fit up to December 2007, we ask how the merger arbitrage strategy would perform given different realizations of the S&P 500 (which happen to correspond to the realized returns for each of the 12 months in 2008). For example, we wish to estimate the expected loss of the merger arbitrage strategy given a 9.21% monthly loss suffered by the S&P 500 (this was the realized return in September 2008, the month of the Lehman Brothers collapse). The steps in this calculation are as follows:

1) Compute the prior probabilities, \( \pi (v \mid \alpha) = \left[ e^{\alpha^T v} / (1 + e^{\alpha^T v}), 1 / (1 + e^{\alpha^T v}) \right] \).

From Table 1, the mixing component contains an intercept term, \( \hat{\alpha}_0 = 1.83 \), and a slope term, \( \hat{\alpha}_1 = 46.18 \). Thus, \( \hat{\alpha}^T = [1.83, 46.18] \). The vector of data is \( v^T = [1, -0.0921] \), where the values reflect the intercept and the given return of the S&P 500, respectively. Then \( \pi (v \mid \alpha) = [0.082, 0.918] \). Note that these values represent the probabilities that the realization was generated from the quiet regime and the hectic regime, respectively.

2) For each regime \( i \), compute the expected loss \( E_i(y \mid x^{(i)}) = \hat{\beta}^{(i)T} x^{(i)} \).

From Table 1, linear component 1 contains the intercept term \( \hat{\beta}^{(1)} = 1.23 \times 10^{-2} \). Set \( x^{(1)} = 1 \), and then \( E_1(y \mid x^{(1)}) = \beta^{(1)T} x^{(1)} = 1.23\% \). Linear component 2 contains intercept and slope terms, respectively: \( \beta^{(2)T} = [-9.24 \times 10^{-3}, 0.29] \). Set \( x^{(2)} = [1, -0.0921] \) and then \( E_2(y \mid x^{(2)}) = -3.57\% \) (keeping additional significant digits).

3) The expected loss is the probability-weighted expected loss conditional on regime:

\[
E(y \mid v, x^{(1)}, x^{(2)}) = \pi_1 E_1(y \mid x^{(1)}) + \pi_2 E_2(y \mid x^{(2)}) = 0.082 (1.23\%) + 0.918 (-3.57\%) = -3.18\%
\]

Table 3 (see page 11) shows the 2008 monthly performance of the S&P 500 index, as well as the monthly expected loss calculations.

Next, we evaluate the out-of-sample performance of the LMLC stress test. Figure 2 (see page 12) presents three pieces of information: 1) a scatterplot showing the 2008 realized monthly returns of the HFRI merger arbitrage index versus the S&P 500 index, 2) expected loss estimates produced by the LMLC model, given S&P 500 index returns between \(-20\%\) and \(5\%\) (solid curve) and 3) expected loss estimates produced by the benchmark model, given S&P 500 index returns between \(-20\%\) and \(5\%\) (dashed curve). For each data point, a vertical line can be drawn to compare how close the model-based estimate was to the realized return of the merger arbitrage index. All other points along each curve represent the hypothetical expected loss of the merger arbitrage index, given the stress setting of the S&P 500 index. For nearly the entire domain, the LMLC model expected losses were...
TABLE 3 LMLC stress test – expected loss computation based on 2008 returns.

| S&P 500 (%)| $\pi_1$ | $\pi_2$ | $E_1(y | x^{(1)})$ (%) | $E_2(y | x^{(2)})$ (%) | ExpectedLoss (%) |
|------------|--------|--------|----------------------|----------------------|-----------------|
| Jan-08     | −6.12  | 0.270  | 0.730                | 1.23                 | −2.68           | −1.63           |
| Feb-08     | −3.48  | 0.556  | 0.444                | 1.23                 | −1.92           | −0.17           |
| Mar-08     | −0.60  | 0.826  | 0.174                | 1.23                 | −1.10           | 0.83            |
| Apr-08     | 4.75   | 0.982  | 0.018                | 1.23                 | 0.44            | 1.22            |
| May-08     | 1.07   | 0.911  | 0.089                | 1.23                 | −0.62           | 1.07            |
| Jun-08     | −8.60  | 0.105  | 0.895                | 1.23                 | −3.40           | −2.91           |
| Jul-08     | −0.99  | 0.798  | 0.202                | 1.23                 | −1.21           | 0.74            |
| Aug-08     | 1.22   | 0.916  | 0.084                | 1.23                 | −0.57           | 1.08            |
| Sep-08     | −9.21  | 0.082  | 0.918                | 1.23                 | −3.57           | −3.18           |
| Oct-08     | −16.83 | 0.003  | 0.997                | 1.23                 | −5.77           | −5.75           |
| Nov-08     | −7.48  | 0.164  | 0.836                | 1.23                 | −3.08           | −2.37           |
| Dec-08     | 0.78   | 0.899  | 0.101                | 1.23                 | −0.70           | 1.04            |

This table shows the calculations involved in the LMLC stress test, based on 2008 realized monthly returns of the S&P 500. $\pi_i$ represents the prior probability that regime $i$ produced the given observation. $E_i(\cdot | x^{(i)})$ is the expected merger arbitrage return conditional on the realized return of the S&P 500, and given that regime $i$ produced the observation. $E_1(\cdot | x^{(1)}) = \beta^{(1)} x^{(1)}$, and, from Table 1, involves only an intercept term: $\hat{\beta}^{(1)} = 1.23 \times 10^{-2}$. $E_2(\cdot | x^{(2)}) = \beta^{(2)} x^{(2)}$ involves an intercept term and a slope term: $\hat{\beta}^{(2)} = [-9.24 \times 10^{-3}, 0.29]$. The expected loss is the inner product of the prior probabilities and the expected returns in each regime, computed as $\text{ExpectedLoss} = \pi_1 E_1(y | x^{(1)}) + \pi_2 E_2(y | x^{(2)})$. Note that negative values indicate losses and positive values indicate gains.

More severe than the benchmark model expected losses. During two of the worst three months for the merger arbitrage index, the LMLC model expected losses were closer than the benchmark model expected losses to the realized returns. This is not surprising, as the LMLC model can better fit the non-linear relationship between merger arbitrage returns and stock market returns than the benchmark model, which is linear. The HitRatio for each model was 71% (five out of seven months correct). The LossUndershoot was 43% and 86% for the LMLC model and benchmark model, respectively. Thus, the LMLC model provided more conservative loss estimates than the benchmark model, based on the realized data.

5 CONCLUSION

This paper outlined the LMLC model and illustrated how it could be used to perform stress testing on an asset that follows regime-switching dynamics. The model is extremely flexible and generalized: the probability that the asset returns are generated from each regime may be determined by a set of stress factors. The linear component within each regime may include a set of distinct stress factors. Once the model is estimated, running a stress test is straightforward; expected loss estimates are produced from the LMLC fitting equation.
FIGURE 2 Model-based expected loss versus realized returns, 2008.

This figure presents the following information: 1) a scatterplot showing the 2008 realized monthly returns of the HFRI merger arbitrage index versus the S&P 500 index, 2) expected loss estimates produced by the LMLC model, given S&P 500 index returns between $-20\%$ and $5\%$ (solid curve) and 3) expected loss estimates produced by the benchmark model, given S&P 500 index returns between $-20\%$ and $5\%$ (dashed curve). For each data point, a vertical line can be drawn to compare how close the model-based estimate was to the realized return of the merger arbitrage index. All other points along each curve represent the hypothetical expected loss of the merger arbitrage index, given the S&P 500 return.

A market stress test was run on a merger arbitrage hedge fund position. It was demonstrated that the LMLC stress test produced expected loss estimates that were more conservative than estimates produced from the benchmark model. In general, the LMLC model will better explain a regime-switching process than linear factor models since there is additional structure to capture hectic periods.

REFERENCES


