

SWING OPTIONS

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ABSTRACT. Swing options are the main type of volumetric contracts in commodity markets. A swing contract gives the holder the right (but not the obligation) to adjust volume of received commodity at her discretion.

Unlike paper assets, trading in physical commodities often takes place over time and therefore involves volume as a second key state variable. Often consumption rates of the commodity are unpredictable and make fixed delivery amounts uneconomic. To mitigate such *volume risk*, a swing contract gives the buyer the opportunity to manage fluctuating commodity demand levels in exchange for a fixed upfront fee. By exercising her swing up/down rights, the buyer can dynamically match supply and demand levels while hedging her costs. Swing options are widely offered by market makers and used extensively by major energy companies, especially in electricity and fossil fuel markets. Contracts with swing features are available both as stand-alone financial tools, and can also be found embedded within structured physical transactions.

1. **Definition and Use.** A typical definition of a swing option is as follows. At times $n = 1, 2, \dots, N$ the buyer is to receive a *baseload* amount \bar{K} of the commodity paying the strike price \bar{P} . In addition, over the life of the contract, the buyer has $N_c \leq N$ swing rights to temporarily vary these deliveries, instead requesting to receive an amount k_n . This request is usually made on a 1-period ahead basis, so that the supplier has time to adjust his delivery. The swing amount k_n is subject to the constraint $k_{min} \leq k_n \leq k_{Max}$, where k_{min} (resp. k_{Max}) is the minimal (resp. maximal) allowable one-period delivery volume. The timing of these exercises is at the discretion of the buyer. Thus, depending on the needs of the buyer, the amount received can be swung up or down up to N_c times.

Let Y_n denote the discounted present value of the net cash flow beyond the income expected from the baseload contract, due to the exercise of a swing right on date n . Assuming that the cashflow amount is linear in the swing volume and no other constraints are present, it is optimal to always apply full (bang-bang) volumetric exercise, i.e. $k_n \in \{k_{min}, k_{Max}\}$. It follows that

$$(1) \quad Y_n = e^{-rn}(k_n - \bar{K})(P_n - \bar{P}),$$

where P_n is the spot price of the commodity in period n , and r is the discount rate for one period. Note that if the exercise decision was made *before* n , then Y_n in (1) may be negative.

Take-or-Pay Provisions. To bound possible swings from the baseload contract, global volume constraints are often imposed. A Take-or-Pay provision adds the constraint $S_{min} \leq \sum_{n=1}^N k_n \leq S_{Max}$, where S_{min} (resp. S_{Max}) is the minimal (resp. maximal) total volume over the N periods. Thus, the buyer is required to place her total cumulative consumption within a pre-defined volume band. This rule is enforced via penalties on the buyer (hence Take-or-Pay) if the provision is violated.

Nomination Contracts. A nomination contract is a variant of a swing option, where the swing amounts are permanent. Thus, changing the nomination amount from \bar{K} to k_n implies that the new baseload amount will be k_n for the remainder of the contract. Such ratcheting rules are common in pipeline contracts.

Refraction Periods. Some contracts impose a minimal refraction period δ between consecutive swings to prevent too rapid an exercise. Refraction periods also appear in research papers that consider swing options in continuous time, see Section 2 below.

Most existing contracts combine several of the above features and also include further custom rules, so that “vanilla” swing options are actually a rarity. Before proceeding with our discussion, we give a few examples of real-life swing contracts.

Swing Option in the Natural Gas Market. Consider an industrial natural gas consumer that wishes to hedge their future input costs. The consumer does not know consumption volume precisely (which depends on demand for output, operational constraints, other suppliers, etc.), but has some volume range based on past usage. The consumer purchases a month-long swing option to offset this “volumetric risk” exposure. The swing option gives them the right to buy up to 10,000 MMBtu at price of $\bar{P} = \$6$ in each of the four weeks; total volume must be between $S_{min} = 10,000$ and $S_{Max} = 30,000$ MMBtu over the month. In this example, $N_c = N = 4$, $k_{min} = \bar{K} = 0$, $k_{Max} = 10000$.

Recall Option for an Oil Pipeline. An oil pipeline company delivers oil from hub A to an industrial customer at point B. The pipeline plans on delivering 1000 bbl/day for the next month at the forward price of $\bar{P} = 100$ \$/bbl. However, due to pipeline congestion, supply disruption or engineering problems, the pipeline may be unable to fulfill its obligation. To mitigate this risk, the pipeline purchases a recall option from its customer. Upon giving notice the day before, the recall option gives the pipeline the right to withhold delivery (or deliver less than the baseload amount) on up to three days during the month. In this example, $N_c = 3$, $N = 30$, $k_{min} = 0$, $\bar{K} = k_{Max} = 1000$. Similar interruptible delivery contracts are common in electricity markets, see [10, 16, 22].

Load Serving Contract for an Electricity Utility. Electricity swing options have become popular with the de-regulation of electricity markets which led to volume risk management by the power serving entities (PSE). A PSE is obligated by law to provide power to its customers and must adjust its production to maintain equilibrium with the fluctuating demand. Such dynamic load management can be synthetically reproduced by a swing contract that the PSE in effect sells to its customers. By buying an offsetting swing option on its input fossil fuel (such as natural gas), the PSE can transfer the corresponding volume risk exposure to other market participants.

2. Valuation and Theoretical Issues. From a financial engineering perspective, a swing option is an exotic compound timing option on the underlying commodity asset. More precisely, it closely resembles a series of American options. Indeed, if the number of swing rights is $N_c = 1$, then the swing collapses to the usual American option [eqf05-007]; if the number of swing rights is equal to the number of periods $N_c = N$, then the value of the swing contract is equal to a strip of European options for each period. Let us denote by $V(t, m, p)$ the value of owning a swing contract on date t with m remaining swing exercise rights and current price p of the commodity. Then the following a priori bound is always true irrespective of the underlying price process:

$$m \cdot EC(t, p) \leq V(t, m, p) \leq m \cdot AC(t, p),$$

where EC (resp. AC) denotes the value of a European (resp. American) option with expiration date N .

In the following discussion we assume that $k_n \in \{0, 1\}$, which up to scaling is equivalent to full volumetric exercise. To setup a mathematical model, it is necessary to construct a framework for the multiple exercise opportunities. Recalling the concept of stopping times, define the set of m -tuples of stopping times used to model the multiple exercises as

$$(2) \quad \mathcal{S}_{\alpha, \beta}^{(m)} = \{\vec{\tau} = (\tau_1, \dots, \tau_m) : \tau_i \text{ stopping time and } \alpha \leq \tau_1 < \tau_2 < \dots < \tau_m \leq \beta\}.$$

Then the optimal multiple stopping problem corresponding to a basic swing option with maturity T reduces to computing

$$(3) \quad V(t, m, p) = \sup_{\bar{\tau} \in \mathcal{S}_{t,T}^{(m)}} \mathbb{E}_t[Y_{\bar{\tau}}], \quad \text{where } Y_{\bar{\tau}} := \sum_{i=1}^m Y_{\tau_i} \quad \text{and } P_t = p.$$

Beyond finding the value of the supremum in (3), it is of interest to know whether this supremum is actually attained (existence of optimal swing exercise strategy), and if yes, finding a computational algorithm giving a “minimal” set of optimal exercise times $\bar{\tau}$. For models of (3) in continuous-time, the refraction time constraint $\tau_i \geq \tau_{i-1} + \delta$, $\delta > 0$ must be added to (2), otherwise $\tau_{i-1} < \tau_i$ is vacuous.

The problem (3) is best solved by an inductive procedure based on the solution of a sequence of single-exercise American options. Namely, [9] have proved that

$$(4) \quad V(t, m, p) = \sup_{t \leq \tau \leq T} \mathbb{E}_t [Y_{\tau} + V(\tau, m - 1, P_{\tau})],$$

where the maximization is over all stopping times τ , and where \mathbb{E}_t denotes conditional expectation at time t . Equation (4) is of the same flavor as the American/Bermudan option problem, but with additional state variable m .

In a Markovian setting, an optimal swing exercise policy is characterized by an *exercise boundary* B , similar to American options. Namely, the exercise boundary divides the commodity price space into a *continuation region*, where the holder of the swing does nothing, and an *exercise region*, where it is optimal to exercise a swing right. However, analysis of the swing exercise boundary is complicated by the fact that today’s exercise reduces future delivery flexibility. Thus, the swing exercise boundary is a function both of time and number of remaining rights, $B = B(t, m)$. In certain settings [6, 9], it can be shown that $B(t, m)$ is connected, and moreover is monotone in number of rights m and similarly is monotone in t . However, general properties of $B(t, m)$ remain unresolved. An excellent summary of the structural features of swing options can be found in the book by Eydeland and Wolyniec [15, Chapter 8].

The major difficulties in valuing and analyzing swing options arise due to the high-dimensional setting and the contract’s path-dependent features. Analysis of a basic swing option involves two state variables, namely the current price P_n and the remaining number of exercise rights M_n . If a Take-or-Pay provision is present, then one must also consider the current cumulative volume $S_n := \sum_{j=1}^n k_j$. Both the M and S state variables depend on the particular swing exercise policy and are therefore path- and control-dependent. This precludes simple application of dynamic programming to (4). Moreover, models of underlying commodity price processes (see [eqf17-013,eqf17-022]) are often complex and include multiple factors to capture the observed seasonality, mean-reversion and nonlinearity in prices. Recall that if the exercise decision is made on a forward basis, modeling of the spot-forward relationship [eqf17-009] is also needed.

To illustrate the difficulties involved, consider a simple binomial tree model that assumes a one-dimensional random walk description of the underlying commodity price (P_n). To price a swing option we then need to set up a *forest* of N_c such trees, with a tree for each possible number of remaining exercise rights. One can then proceed using the backward dynamic programming algorithm like for standard American options [eqf05-007], where a swing exercise triggers a jump from a tree with $M_n = m$ remaining rights to a tree with $M_n = m - 1$. Such an algorithm imposes memory and space constraints and raises efficiency issues. More sophisticated models might involve multiple factors or jumps (or Take-or-Pay provisions) and would require even more complex forests of trees.

3. Historical Perspective. Initial discussion of swing options has appeared in early 1990s in energy practitioner magazines, see e.g. [3, 11, 12]. At the same time a limited literature studied the embedded multiple exercise opportunities [30, 25, 14]. Theoretical treatment of swing options was first carried out rigorously by [21]. The connection to multiple stopping problems was established by Carmona and Touzi [9], who showed in a certain mathematical framework that a swing option is equivalent to N_c compound American (optimal stopping) options on the underlying, see (4). This fact suggests pricing of swing options based on existing methods for American options. In a simplified model with single underlying risky asset, swing options could be for example priced by solving partial differential equations like in the classical Black- Scholes theory [eqf12-002]. However, non-linearities rule out solutions in the classical sense, and complex free boundaries make the design and control of numerical schemes quite a challenge.

To overcome these challenges, a variety of approaches have been suggested, including other partial differential equation methods ([13, 31] and [eqf12-007]), stochastic dynamic programming ([2, 17]), the aforementioned binomial and trinomial trees ([21] and [eqf12-017]), Monte Carlo simulation methods ([27, 4, 20, 16] and [eqf13-006]), approximate techniques ([14, 23, 29] and [eqf13-024]) and optimal quantization ([1]). Also see [16, 19, 24, 32] for analytic pricing of swing contracts under certain exotic models for the underlying commodity and [28] for game-theoretic aspects of swing options. Beyond the basic contracts, the modern practitioner/research trend is to value swing options using simulation tools which can achieve excellent computational complexity properties with respect to underlying models. However, as a trade-off, the error analysis of many simulation schemes is difficult, and usually only a lower bound on price can be obtained.

4. Hedging Issues. Hedging of swing options can be carried out in the same framework as American options. However, several additional issues make hedging difficult in practice. First, recall that many buyers of swing options are industrial customers. Empirical evidence [15] suggests that such buyers often exercise sub-optimally in order to meet their other obligations (e.g. as suppliers of electricity). Sub-optimal exercise may be justified if the spot market is not sufficiently liquid, as is the case in many commodities, especially those with strong locational characteristics. This leads to non-trivial financial implications to sellers of swing options who hedge optimally.

Secondly, typically swing exercise decisions are made day-ahead on the basis of forward prices. Since the cashflow is based on spot prices, basis risk between forward and spot price becomes of concern (both to buyer and seller of swing contracts). This is particularly so in electricity markets, where the commodity is non-storable and there is no definite convergence between day-ahead and real-time prices.

The underlying commodity market may be financially incomplete [eqf04-006] for other reasons. As such, the seller of the swing option cannot fully hedge her exposure and therefore the no-arbitrage risk-neutral valuation method might not make sense. Finally, in some less liquid markets, the buyer may be exercising market power by manipulating demand through her swing policy and therefore affecting future prices.

5. Extensions and other Contracts with Multiple Exercise. With wide use of swing options, other modifications to the basic contract have become common.

Chooser Flexible Caps. A cap contract with strike \bar{P} on a 3-months interest rate is a portfolio of options on the quarterly interest payments that have to be made. Let us assume that the life of the contracts is N quarters, and the nominal is X . Each individual option is called a caplet. It covers one of the N quarters, and its payoff is $X \cdot \Delta t \cdot (L_3 - \bar{P})^+$, where Δt is the daycount fraction, and L_3 denotes the quarter-end 3-month LIBOR rate. A *chooser flexible cap* is a contract such that at each reset date, the holder of the contract has the right to decide whether to exercise that particular caplet and count it as part of the N_c allowable ones, or spare it for later use, each decision being

final. The remaining caplet rights expire worthless if not used before the end of the N periods. A chooser cap reduces to a regular cap [eqf11-006] when $N_c = N$, while it can be viewed as a standard American/Bermudan option on the interest rate L_3 when $N_c = 1$. Valuation of such contracts was studied in [27].

Two Sided Swing Options. Some physical contracts combine the swing and recall features described above. Thus, the baseload contract is augmented with N_r recall rights for the party delivering the commodity, and N_c swing rights for the buyer. The interaction of these rights leads to a stochastic game between the two counter-parties, with the buyer being the minimizing agent, and the seller the maximizing agent. Valuation of such two-sided swing options is related to Dynkin games as explained in [5].

Gas Storage. An owner of a gas storage facility attempts to maximize revenue by injecting gas into storage when prices are low and withdrawing gas when prices are high. Exercise of such flexibility can be viewed as a nomination swing option where k_n is allowed to be both positive and negative (to model withdrawals). The volume constraints $S_{min} \leq S_n \leq S_{Max}$ then correspond to the physical size constraints of the facility. Gas storage valuation is complicated by additional storage costs which add further path-dependency to the problem, see [4, 8].

Tolling Agreements. In a *tolling agreement*, the buyer is granted use of a commodity generating facility for a fixed period of time. The buyer can then run the facility when commodity price is high and keep it shut down when prices are low. This is again a nomination swing option with additional engineering constraints, such as fixed switching costs. Modeling of such contracts fits within the more general framework of optimal switching problems ([7, 18]).

Employee Stock Options. Company employees are often granted multiple stock options in their employers [eqf03-015]. Partial exercise of such options by risk-averse employees is similar to a swing call option, as multiple exercises must be considered ([26]).

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