UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Statistics and Applied Probability
PSTAT 221, Advanced Probability Theory, Fall 2008
Syllabus

Instructor: Dr. János Engländer
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Preliminary office hours:
Tuesday 11-12
Thursday 11-12

Goal of the course: Diffusion processes play a central role in probability theory and its applications, notably in mathematical finance and spatial population models. They are also intimately related to elliptic and parabolic partial differential equations.
The goal of the course is to give a firm and rigorous background on the construction and basic properties of diffusion processes, presenting both analytic and probabilistic techniques.

Prerequisites: This is an advanced course and so the students are supposed to be familiar with measure theoretic probability, Brownian motion, Itô integrals, Itô formula and SDE's, although we may briefly review some facts in these topics.
For example, being familiar with the first five chapters of Øksendal's book (see citation nr. 4 below) is sufficient background before starting this class.

Course web site: `http://www.pstat.ucsb.edu/faculty/englander/221A.html`

Course content[1]**:**

- **Existence and uniqueness for diffusion processes on $\mathbb{R}^d$ and on Euclidean domains $D \subset \mathbb{R}^d$.** The martingale problem and the generalized martingale problem on $\mathbb{R}^d$ and on $D \subset \mathbb{R}^d$. The Girsanov transform.
- **Basic properties of diffusion processes.** Stochastic representation for solutions of elliptic equations. Exit time from a small ball. Blumenthal 0-1 law. Regularity of boundary points. Feynman-Kac formula. Stroock-Varadhan Support Theorem.
- **Further properties/examples of diffusion processes.** Diffusion with killing. The Bessel process. Itô process versus diffusion. Random time change. Angular part of Brownian motion. Brownian motion on the unit sphere.
- **Some analytic properties.**The h-transform (of Doob), the Green's measure and Green's function.
- **One-dimensional diffusions.** Regular diffusions. Scale function and speed measure, local time, classification of boundary points. Explosion, ergodicity. Coupling.

Textbooks:

- *Positive Harmonic Functions and Diffusion* (Cambridge Studies in Advanced Mathematics) by Ross G. Pinsky
- *Diffusions, Markov Processes, and Martingales: Volume 1-2*, (Cambridge Mathematical Library) by L. C. G. Rogers and David Williams

[1]Depending on the pace and interest, we might add some more topics from diffusion theory

Further literature:

1. *Brownian Motion and Stochastic Calculus* (Graduate Texts in Mathematics) by Ioannis Karatzas and Steven E. Shreve
2. *Multidimensional Diffusion Processes* (Classics in Mathematics) by Daniel W. Stroock and S.R.S. Varadhan
3. *Partial Differential Equations for Probabilists* (Cambridge Studies in Advanced Mathematics) by Daniel W. Stroock
4. *Stochastic Differential Equations: An Introduction with Applications* (Universitext) by Bernt Øksendal
5. *Stochastic Differential Equations and Applications* (Dover Books on Mathematics) by Avner Friedman

Homework: Homework will be given regularly. You will have to submit them in time. However you may have the opportunity to make corrections.

Office hours: Please come to office hours to ask questions and provide feedback on the class.

Course grades: will be based on homework.

Enjoy the course!