Correction to Problem # 2

There was a mistake in the solution to problem # 2 from assignment 7. Here is an updated solution.

(b)

The probability that Bob wins the remaining two games and thus brings his total to 3 is $(1-p)^2$ if we know p (or conditional on p). The estimator of this quantity can be calculated by taking the posterior expectation

$$\begin{aligned}
\mathbb{E}((1-p)^2 \mid X=2) &= \int_0^1 (1-p)^2 \frac{1}{B(3.2, 1.8)} p^{2.2}(1-p)^{0.8}\, dp \\
&= \frac{1}{B(3.2, 1.8)} \int_0^1 p^{2.2}(1-p)^{2.8}\, dp \\
&= \frac{B(3.2, 3.8)}{B(3.2, 1.8)} \\
&= \left(\frac{\Gamma(3.2)\Gamma(3.8)}{\Gamma(7)}\right)\left(\frac{\Gamma(5)}{\Gamma(3.2)\Gamma(1.8)}\right) \\
&= \left(\frac{\Gamma(3.8)}{\Gamma(1.8)}\right)\left(\frac{\Gamma(5)}{\Gamma(7)}\right) \\
&= \frac{(2.8)(1.8)}{5(6)} = 0.168
\end{aligned}$$

In order to calculate the Γ functions, I have used the following identity

$$\frac{\Gamma(\alpha+1)}{\Gamma(\alpha)} = \alpha$$

for any $\alpha > 0$. It follows almost directly that

$$\frac{\Gamma(\alpha+2)}{\Gamma(\alpha)} = \left(\frac{\Gamma(\alpha+2)}{\Gamma(\alpha+1)}\right)\left(\frac{\Gamma(\alpha+1)}{\Gamma(\alpha)}\right) = (\alpha+1)\alpha.$$

(c)

This new answer for part (b) means that the answer for part (c) must be

$$\$60(1-0.168) + \$40(0.168) = \$56.64$$