Review questions

1. What are the two formulas for the minimum of exponential random variables, and where did we use them later?

2. Give two interpretations for the rate of a Poisson process (**PP**)(Hint: one is a probability, the other a mean)

3. What are the conceptual differences between the two equivalent definitions of a **PP**?

4. For a homogeneous **PP**, what is the distribution of the inter-arrival times? What is their conditional distribution given there are k jumps by time t?

5. If a **PP** is marked randomly with different colors (with constant probabilities over time), what are the properties of the resulting processes?

6. In **5.**, what if the probabilities depends on time?

7. What is the difference between a non-homogeneous and a homogeneous **PP**?

8. What is a M/G/1 queue?

9. What is a compound **PP**? Give also an alternative representation.

10. Give an informal description of a continuous time Markov Chain (**MC**).

11. Why is the waiting time exponential in a **MC**?

12. What are the "ingredients" for a **MC**. Give two description (one in terms of rates, one in terms of probabilities)

13 Describe a Birth and Death process (**B&D**). Again gives two (equivalent) description, one in terms of rates, one in terms of probabilities.

14 Write a $M/M/\infty$ queue as a **B&D**.

15 Write a $M/M/k$ queue as a **B&D**.

16. Give the definition of the transition probability function of a **MC** and state the Chapman-Kolmogorov equation. Give both the developed and compact forms (the later in terms of the matrix Q).

17. Write down the backward equation for a **B&D**.

18. Write down the "balance equation" for the limiting probabilities for a general **MC**, and then for the special case of a **B&D**.

19. Give the general formula for the limiting probabilities for a **B&D**, and then for the special case of a $M/M/1$ queue. Check carefully the conditions for the validity of the formula and give an intuitive explanation.

20. What is the correct scaling (step size vs step time) of a random walk in order to converge to the Brownian motion, and why is it the only valid one?

21. Give the definition of the Brownian motion.

22. Explain briefly how the reflection principle can be used to find the distribution of the maximum.